Worked examples for the design of steel structures
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Worked examples for the design of steel structures

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Part 1.1 General rules and rules for buildings
(together with United Kingdom National Application Document)
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Foreword

This Publication has been funded and prepared jointly by the Steel Construction Institute (SCI), Ove Arup & Partners, the Building Research Establishment (BRE) and the Department of the Environment (DOE), to promote and assist the use of British Standard DD ENV 1993-1-1: 1992, Eurocode 3: Design of steel structures* Part 1.1 General rules and rules for buildings (together with United Kingdom National Application Document).

The worked examples have been prepared in accordance with Eurocode 3 and make use of design aids contained in C-EC3: Concise Eurocode 3 for the design of steel buildings in the United Kingdom, published by the SCI.

Attention is drawn to Approved Document A (Structure) in support of The Building Regulations 1991, which states that Eurocode 3, together with the National Application Document, provides appropriate guidance for the design of steel buildings in the United Kingdom.

Technical enquiries should be addressed to either the Building Research Establishment or the Steel Construction Institute.

* A DD ENV is a British Standard implementation of the English-language version of a European Pre-Standard (ENV), published as a Draft for Development (DD).
Introduction

This book provides engineers and students with a set of examples that meet the requirements of British Standard DD ENV 1993-1-1: 1992, Eurocode 3: Design of steel structures Part 1.1 General rules and rules for buildings (together with United Kingdom National Application Document)\(^1\).

The examples include a 5-storey steel-framed building and five other steel structures. Each example has been prepared to give a detailed indication of the process of designing steel structures to Eurocode 3, including all the checks required by the Eurocode and the UK National Application Document (NAD).

Supplementing DD ENV 1993-1-1: 1992, the Steel Construction Institute has produced C-EC3, a concise version of Eurocode 3: Part 1.1 in a form familiar to engineers in the United Kingdom\(^2\). Where appropriate, the use of this concise document is highlighted.

Marginal notes show the appropriate reference in either Eurocode 3: Part 1.1, the UK National Application Document, the concise document or the British Standards. They are given as follows:

**Eurocode 3: Part 1.1**
- Clauses 2.2.2.2
- Tables Table 2.2
- Figures Figure 5.3.2
- Equations Equation 2.11

**National Application Document**
- Clauses NAD 6.1
- Tables NAD Table 1

**Concise document (C-EC3)**
- Clauses C-EC3 6.5.3
- Tables C-EC3 Table 6.5

**British Standards**
- BS 5573: Part 3: 1983
  - Clause 3.1.4

Generally, the solutions presented in this publication are aimed at illustrating the economic design of steel. However, it must be emphasised that the examples have been chosen to demonstrate specific requirements in Eurocode 3 and the NAD. Consequently, alternative solutions may exist which more closely reflect standard fabrication practice, and which provide greater overall economy.

All the examples have been prepared on the basis of the product standards for steel material current at the time the work was done; for example British Standard BS EN 10025:1990\(^3\) generally, but British Standard BS 4360:1990\(^4\) for hollow sections, i.e Fe 430 for a UB but grade 43 for a CHS.

Since then, British Standard BS EN 10025:1993 has been issued and British Standard BS EN 10210 is expected to be issued soon. In these two Standards the equivalent grade to Fe 430 and 43 has become S275 in both cases.

It should be noted that the axis notation used in Eurocode 3: Part 1.1 differs from that used in the UK. The y-y axis is the major axis and the z-z axis is the minor axis (see Figure 1.1 in the Eurocode). Extreme care should be taken when conducting designs to Eurocode 3: Part 1.1 and when using existing published section data.

Figure 1.1
Tabulated section data, conforming to the new axis notation and introducing properties specific to Eurocode 3: Part 1.1, can be found in section tables.

It should also be noted that in Eurocode 3: Part 1.1 the throat thickness is used to specify a fillet weld, rather than the leg length.

The best way to familiarise oneself with the Eurocode is to use it in actual design, and the authors hope that with the aid of these examples engineers will soon gain the experience to design economic structures to the Eurocode.

Users of DD ENV 1993-1-1: 1992 are invited to comment on its technical content, ease of use, and any ambiguities or anomalies. These comments will be taken into account during preparation of the UK national response to the European Committee for Standardization (CEN) on the question of whether the ENV (Pre-Standard) can be converted to an EN (full Standard).

Comments should be sent in writing to the British Standards Institution, 2 Park Street, London W1A 2BS, quoting the document reference, the relevant clause and, where possible, a proposed revision.
Example 1
Design of a 5-storey braced frame

1.1 Frame geometry

This chapter covers the design of a 5-storey braced steel-framed building. In particular, it gives detailed designs for the primary and secondary floor beams, a transfer plate girder carrying column loads, an internal column, and a number of different connection types.

The geometry of the building reflects modern composite construction practice. However, the benefits of composite action have been neglected. Composite design is dealt with separately in Eurocode 4, scheduled for publication in 1994.

Figure 1 shows details of the 5-storey building, representing a small, 4-storey office development constructed over a showroom.

Figure 1  Typical cross-section (dimensions in mm)
Figure 2 shows a typical part plan. Details of the construction are as follows:

**Construction**
- Flat roof: Asphalt on 130 mm lightweight concrete on profiled metal decking
- Floors (office use): Raised floor on 130 mm lightweight concrete on profiled metal decking
- External walls: Proprietary cladding
- Fire protection: 4-hour fire rating between ground floor and 1st floor, 2-hour fire rating between 1st floor and roof
- Bracing: The building is braced against side-sway

![Figure 2](typical_part_plan.png)

*Figure 2: Typical part plan (dimensions in mm)*

**Design assumptions**
In conformity with typical multi-storey steel-frame construction in the UK, it is assumed that resistance to lateral wind loads is provided by a system of localised cross-bracing, and that the main steel frame is designed to support gravity loads only.

The connections are designed to transmit vertical shear, and to be capable of transferring a horizontal tying force to preserve the integrity of the structure in the event of accidental damage. It is also assumed that the connections offer little, if any, resistance to free rotation of the beam ends.

With these assumptions, the frame is classified as ‘simple’, and the internal forces and moments are determined using a global analysis which assumes the members to be effectively pin-connected.

Until publication of the loading Eurocode, all loading should be assessed using the loading codes shown in the NAD.

Suitable methods for designing columns in simple framed structures are given in Annex B of the NAD.
1.2 Loading

Permanent actions
The weights of building materials are given in British Standard BS 648: 1964 Schedule of weights of building materials⁸.

<table>
<thead>
<tr>
<th>Typical floor</th>
<th>kN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raised floor (manufacturer’s literature)</td>
<td>0.2</td>
</tr>
<tr>
<td>130 mm lightweight concrete on profiled metal decking</td>
<td>2.5</td>
</tr>
<tr>
<td>Steelwork and fire protection</td>
<td>0.5</td>
</tr>
<tr>
<td>Services</td>
<td>0.3</td>
</tr>
<tr>
<td>Ceiling</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\[ G_{k,1} = 3.7 \]

<table>
<thead>
<tr>
<th>Roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paving and insulation</td>
</tr>
<tr>
<td>Asphalt</td>
</tr>
<tr>
<td>130 mm lightweight concrete on profiled metal decking</td>
</tr>
<tr>
<td>Steelwork and fire protection</td>
</tr>
<tr>
<td>Services</td>
</tr>
<tr>
<td>Ceiling</td>
</tr>
</tbody>
</table>

\[ G_{k,2} = 5.0 \]

Cladding

Proprietary cladding                 | 0.8   |

\[ G_{k,3} = 0.8 \]

Variable actions
Variable actions for buildings are given in the following British Standards:


Floor loads⁸

<table>
<thead>
<tr>
<th>Imposed load (client’s brief)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(BS 6399: Part 1 requires 2.5 kN/m² for offices⁸)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Allowance for metal partitions not shown on plans</th>
</tr>
</thead>
</table>

\[ Q_{k,1} = 5.0 \]

Roof loads⁸

<table>
<thead>
<tr>
<th>Imposed load for roof with access</th>
</tr>
</thead>
<tbody>
<tr>
<td>(This is significantly greater than snow load which need not, therefore, be considered)</td>
</tr>
</tbody>
</table>

\[ Q_{k,3} = 1.5 \]

Wind loads⁹

<table>
<thead>
<tr>
<th>From British Standard CP 3, dynamic wind pressure, q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Characteristic dynamic wind pressure, Q_{k,3} = 0.9 \times 0.76</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.68</td>
</tr>
</tbody>
</table>

References

NAD 4

BS 648: 1964
British Standard CP 3: Chapter V: Part 2*, Table 10 gives the following force coefficients, $C_f$, for a building with $l/w = 3.0$ and height/breadth $= 1.2$:

- Transverse wind: 1.2
- Longitudinal wind: 0.75

**Ultimate limit states**
The partial safety factors for ultimate limit states are:

*Permanent actions*

\[ \gamma_{G,up} = 1.35 \text{ for unfavourable effects} \]

*Variable actions*

\[ \gamma_{Q,up} = 1.5 \text{ for unfavourable effects} \]

This structure is classified as a simple frame, and therefore pattern loading of imposed loads need not be considered.

**Serviceability limit states**
For deflection calculations the rare combination is used, so in this case the design loads for the serviceability limit state are equal to the specified loads.
1.3 Fully restrained beam (B1)
The secondary beam (B1) shown in Figure 3 is simply supported at both ends and is fully restrained along its length.

For the loading shown, design the beam in grade Fe 430 steel, assuming that it is carrying plaster or a similar brittle finish.

![Diagram of fully restrained beam](image)

Figure 3 Fully restrained beam

1.3.1 Loading

Characteristic values

Variable action \( Q_k = 5.0 \times 2.5 \text{ kN/m} = 12.5 \text{ kN/m} \)
Permanent action \( G_k = 3.7 \times 2.5 \text{ kN/m} = 9.25 \text{ kN/m} \)

The permanent action includes an allowance for the weight of the beam and casing.

Partial safety factors

Variable action \( \gamma_{Q,sup} = 1.5 \)
Permanent action \( \gamma_{G,sup} = 1.35 \)

1.3.2 Design values

\[
F_d = \gamma_{G,sup} G_k + \gamma_{Q,sup} Q_k \\
= 1.35 \times 9.25 + 1.5 \times 12.5 = 31.2 \text{ kN/m}
\]

Design moment

\[
M_{sd} = \frac{F_d L^2}{8} = \frac{31.2 \times 7.52^2}{8} = 219 \text{ kNm}
\]

Design shear force

\[
V_{sd} = \frac{F_d L}{2} = \frac{31.2 \times 7.52}{2} = 117 \text{ kN}
\]

To determine the section size, assume that the flange thickness is less than 40 mm so that the design strength is 275 N/mm², and that the section is class 1 or 2.

\[
M_{sd} \leq M_{e,Rd}
\]

whence, for class 2 section,

\[
M_{e,Rd} = M_{p,fy,Rd} = W_p f_y / \gamma_{M0}
\]

\[
W_{p,required} = M_{sd} \gamma_{M0} f_y \\
= (219 \times 10^3 \times 1.05)/275 = 836 \text{ cm}^3
\]
Try a $406 \times 140 \times 46$ UB

Section properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth $h$</td>
<td>402.3 mm</td>
</tr>
<tr>
<td>Width $b$</td>
<td>142.4 mm</td>
</tr>
<tr>
<td>Web thickness $t_w$</td>
<td>6.9 mm</td>
</tr>
<tr>
<td>Flange thickness $t_f$</td>
<td>11.2 mm</td>
</tr>
<tr>
<td>Depth between fillets $d$</td>
<td>359.7 mm</td>
</tr>
<tr>
<td>Plastic modulus $W_{fr}$</td>
<td>888 cm$^3$</td>
</tr>
</tbody>
</table>

This notation conforms with Figure 1.1 in Eurocode 3: Part 1.1.

### 1.3.3 Classification of cross-section

As a simply supported beam is not required to have any plastic rotation capacity (only one hinge required), it is sufficient to ensure that the section is at least class 2 to develop the plastic moment resistance.

Figure 4 shows a typical cross-section for a universal beam.

![Figure 4 Typical cross-section](image)

**Flange buckling,** $c/t_f \leq 11 \varepsilon$

where $c$ = half the width of the flange

$t_f$ = the flange thickness

(if the flange is tapered, $t_f$ should be taken as the average thickness)

$$\varepsilon = \sqrt{(235/t_f)} = \sqrt{(235/275)} = 0.924$$

For this section the limit is $11 \varepsilon = 11 \times 0.924 = 10.2$

$$c/t_f = \frac{142.4}{2 \times 11.2} = 6.36$$

**Web buckling,** $d/t_w \leq 83 \varepsilon = 83 \times 0.924 = 76.7$

where $d$ = depth between root radii

$t_w$ = web thickness

$$d/t_w = \frac{359.7}{6.9} = 52.1$$

$c/t_f < 11 \varepsilon$ and $d/t_w < 83 \varepsilon$

$\therefore$ section is at least class 2.
1.3.4 Deflection check

Eurocode 3 requires that the deflections of the beam be checked under the following serviceability loading conditions:

- Variable actions, and
- Permanent and variable actions.

Figure 5 shows the vertical deflections to be considered.

![Diagram of vertical deflections]

Figure 5  Vertical deflections

\[ \delta_0 \] is the precamber
\[ \delta_1 \] is the deflection due to permanent action
\[ \delta_2 \] is the deflection caused by variable actions, and
\[ \delta_{\text{max}} \] is the total deflection caused by permanent and variable actions less any precamber.

For a plaster or similar brittle finish, the deflection limits are \( L/250 \) for \( \delta_{\text{max}} \) and \( L/350 \) for \( \delta_2 \). Deflection checks are based on the serviceability loading.

For a uniform load

\[ \delta = \frac{5}{384} \times \frac{F_k L^4}{EI_y} \]

where
- \( F_k \) is the total load = \( Q_k \) or \( (G_k + Q_k) \) as appropriate
- \( L \) is the span
- \( E \) is the modulus of elasticity (210 000 N/mm²)
- \( I_y \) is the second moment of area about the major axis (y–y)

For unit load of 1 kN/m

\[ \delta = \frac{5}{384} \times \frac{10^3 \times 7.5 \times 7500^3}{210 000 \times 15 600 \times 10^4} = 1.3 \text{ mm} \]

The calculated deflections shown in Table 1 are less than the limits, so no pre-camber is required. It should be noted that if the structure is open to the public, there is a limit of 28 mm for the total deflection of \( \delta_1 + \delta_2 \) (neglecting any precamber) under the frequent combination, to control vibration. For the frequent combination the variable action is multiplied by \( \psi_1 \) which has a value of 0.6 for offices.

4.3.2 (2)

Table 1  Calculated and limiting deflections

<table>
<thead>
<tr>
<th>Deflection limit (mm)</th>
<th>Calculated deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_1 ) permanent action</td>
<td>1.3 × 9.3 = 12.1</td>
</tr>
<tr>
<td>( \delta_2 ) variable action</td>
<td>1.3 × 12.5 = 16.3</td>
</tr>
<tr>
<td>( \delta_{\text{max}} )</td>
<td>28.4</td>
</tr>
<tr>
<td>L/250 = 21.4</td>
<td></td>
</tr>
<tr>
<td>L/250 = 30.0</td>
<td></td>
</tr>
</tbody>
</table>
1.3.5 Shear on web

The shear resistance of the web must be checked.

\[ V_{sl} \leq V_{p/rd} \]

The design plastic shear resistance of the web is given by:

\[ V_{p/rd} = \frac{f_{y} \sqrt{3}}{\gamma_{M0}} \]

For rolled I and H sections loaded parallel to the web,

shear area \( A_{v} = 1.04 \cdot t_{w} \)

and the partial safety factor \( \gamma_{M0} = 1.05 \)

\[ V_{p/rd} = \frac{1.04 \cdot t_{w} \cdot f_{y}}{\sqrt{3} \cdot \gamma_{M0}} \]

\[ V_{p/rd} = \frac{1.04 \times 402.3 \times 6.9 \times 275}{\sqrt{3} \times 1.05 \times 10^3} = 437 \text{ kN} \]

This is greater than the shear on the section (117 kN).

As this beam has partial depth end-plates, a local shear check is required on the web of the beam where it is welded to the end-plate.

\[ V_{p/rd} = \frac{f_{y} \sqrt{3}}{\gamma_{M0}} \]

where \( A_{v} = t_{w} \cdot d \)

\[ d = \text{depth of end-plate} = 250 \text{ mm (see also Figure 18 in Section 1.9.2)} \]

\[ V_{p/rd} = \frac{6.9 \times 250 \times 275}{\sqrt{3} \times 1.05 \times 10^3} = 260.8 \text{ kN} \]

This is greater than the shear on the section (117 kN).

A further check is sometimes required, especially when there are significant point loads, cantilevers or continuity, to ensure that the shear will not have a significant effect on the moment resistance. This check is carried out for the moment and shear at the same point. The moment resistance of the web is reduced if the shear is greater than 50% of the shear resistance of the section. With a uniform load, the maximum moment and shear are not coincident and this check is not required for beams without web openings.

1.3.6 Additional checks if section is on seating cleats (etc)

In this example the beam has partial depth end-plates. There are, however, cases where the beams may be supported on seating cleats, or on other materials such as concrete pads. A similar situation arises when a beam supports a concentrated load applied through the flange. In these cases, make the following checks:

- Crushing of the web
- Crippling of the web
- Buckling of the web

References

5.4.6

NAD Table 1

5.4.7.3

5.7.3

5.7.4

5.7.5
The following detailed checks are for a 75 mm stiff bearing.

**Crushing resistance**
The crushing resistance is given by:

\[
R_{y, Rd} = \frac{(s_y + s_x) t_w f_{yw}}{\gamma_M}
\]

where
- \(s_x\) is the length of the stiff bearing (75 mm)
- \(t_w\) is the web thickness
- \(f_{yw}\) is the yield strength of the web
- \(\gamma_M\) is the material partial safety factor (1.05)
- \(s_y\) is the length over which the effect takes place, based on the section geometry and the longitudinal stress in the flange

\[
s_y = 2 t_f (b / A_w)^{0.55} (f_{yf} f_{yw})^{0.55} \left[1 - \left(\frac{\sigma_{tEd}}{f_{yf}}\right)^2\right]^{0.55}
\]

At the support, the stress in the beam flange, \(\sigma_{tEd}\), is zero, \(f_{yf} = f_{yw}\) but the value of \(s_y\) is halved

\[
s_y = \frac{2 \times 11.2 \times (142.4/6.9)^{0.55}}{2} = 51\text{ mm}
\]

\[
\therefore \text{crushing resistance } = \frac{(75 + 51) \times 6.9 \times 275}{1.05 \times 10^3} = 227.7\text{ kN}
\]

This is greater than the reaction (117 kN).

**Crippling resistance**
The crippling resistance is given by:

\[
R_{s, Rd} = \frac{0.5 t_w^2 (E f_{yw})^{0.55} [(t/2)^{0.55} + 3(t_w/t_d)(s_e/d)]}{\gamma_M}
\]

\(s_e\) is limited to a maximum of 0.2 \(d\) (402.3 \(\times\) 0.2 mm = 80 mm)

\[
R_{s, Rd} = 0.5 \times 6.9^2 (210 000 \times 275)^{0.55} [11.2/(6.9)^{0.55} + 3(6.9/11.2)(75/359.5)](1.05 \times 10^3)
\]

\[
= 286\text{ kN}
\]

This is greater than the reaction (117 kN).

**Buckling resistance**
The buckling resistance is determined by taking a length of web as a strut.
The length of web is taken from Eurocode 3 which, in this case, gives a length:

\[
d_{eff} = 0.5 (h^2 + s_e^2)^{0.55} + a + \frac{s_e}{2} \text{ but } \leq (h^2 + s_e^2)^{0.55}
\]

where
- \(a = 0\)
- \(b_{eff} = 0.5 (402.3^2 + 75^2)^{0.55} \times \frac{75}{2} = 242\text{ mm}\)

Provided that the construction is such that the top flange is held by a slab and the bottom by seating cleats, against rotation and displacement, the effective height of the web for buckling should be taken as 0.7 \(\times\) distance between fillets.

\[
\ell' = 359.7 \times 0.7\text{ mm } = 252\text{ mm}
\]
Radius of gyration for web
\[ i = d/\sqrt{12} = 0.29 \times d \]

Slenderness \( \lambda = \ell i \frac{252}{0.29 \times 6.9} = 126 \)
\[ \lambda = \lambda_i (\beta_A)^{0.5} \]

where \( \lambda_i = 93.9 \times e = 93.9 \times 0.924 = 86.8 \)
\[ \lambda = \frac{126}{86.8} = 1.45 \]

Using buckling curve c and \( \beta_A = 1.0 \), the value of \( \chi \) may be determined from Table 5.5.2
\[ \chi = 0.33 \]

Buckling resistance \( N_{b,rd} = \frac{0.33 \times 275 \times 242 \times 6.9}{1.05 \times 10^3} = 144 \text{kN} \)

This is greater than the reaction (117 kN),
\[ \therefore \text{satisfactory.} \]

1.3.7 Summary
The trial section 406 \times 140 \times 46 UB is satisfactory if the section is on a stiff bearing 75 mm long. If it is supported by web cleats or welded end-plates, the web checks, except for shear, are not required and the section is again satisfactory.

1.3.8 Design procedure using the concise document (C-EC3)\(^2\)
This beam can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following checks, in which a simpler procedure is used.

Web buckling resistance
The procedure for determining buckling resistance has been simplified by using a buckling strength, \( f_c \), based on \( \lambda \) and not \( \lambda \).
\[ N_{b,rd} = \beta_A f_c A/\gamma_{M1} \]
where
\[ A = t_w b_{eff} \]
\[ t_w = 6.9 \text{mm} \]
\[ b_{eff} = 242 \text{mm} \]
\[ \beta_A = 1 \]
\[ \gamma_{M1} = 1.05 \]
\[ \lambda = 127 \]
\[ \therefore \sqrt{(\beta_A)} \lambda = 127 \]
Using buckling curve c:
\[ f_c = 90 \text{ N/mm}^2 \]

\[ \therefore N_{b, Rd} = \frac{1.0 \times 90 \times 6.9 \times 242}{1.05 \times 10^3} = 143 \text{ kN} \]

This is greater than the reaction (117 kN),

\[ \therefore \text{satisfactory.} \]
1.4 Beam restrained at load points (B2)
The primary beam shown in Figure 6 is laterally restrained at the ends and at
the points of application of the load. For the loading shown, design the beam
in grade Fe 430 steel.

![Beam B2 restrained at load points (dimensions in mm)](image)

Figure 6  Beam B2 restrained at load points (dimensions in mm)

1.4.1 Loading
Characteristic values
The point loads are taken as the end reactions from beams B1 (see Section 1.3).

Variable action at point load $Q_{k1}$  
= $5.0 \times 2.5 \times 7.5$
= $94 \text{kN}$

Permanent action at point load $G_{k1}$  
= $3.7 \times 2.5 \times 7.5$
= $69 \text{kN}$

For self-weight of beam B2 and casing, allow  
$G_{k2} = 12.5 \text{kN}$

Partial safety factors
Variable action \( \gamma_{Q,	ext{sup}} = 1.5 \)  
Permanent action \( \gamma_{G,	ext{sup}} = 1.35 \)

1.4.2 Design values
Point loads
The design value per point load is:

\[
F_d = \gamma_{G,	ext{sup}} G_{k1} + \gamma_{Q,	ext{sup}} Q_{k1}
\]

\[
F_{d1} = 1.35 \times 69 + 1.5 \times 94
= 234 \text{kN}
\]

Self-weight
The self-weight of the beam and casing are assumed to be uniformly
distributed along the full length of the beam.

\[
F_{d2} = 12.5 \times 1.35
= 17 \text{kN}
\]

Reactions
\[
V_{d1} = F_{d1} + F_{d2}/2
V_{d2} \text{ (at supports)} = 234 + 17/2 = 242.5 \text{kN}
\]
Design moment
Figure 7 shows the distribution of bending moments.

![Load points diagram](image)

Maximum moment = 601 kNm

Figure 7 Bending moment diagram

Moment at mid-span (maximum)

\[ M_{SL} = 234 \times 2.5 + 17 \times 7.5/8 = 601 \text{ kNm} \]

Moment at load point

\[ M_{SL} = 242.5 \times 2.5 + 17/7.5 \times 2.5^2/2 = 599 \text{ kNm} \]

1.4.3 Initial section selection

Assume that a rolled universal beam will be used and that the flanges will be less than 40 mm thick. For grade Fe 430 steel, \( f_y = 275 \text{ N/mm}^2 \). Because the beam is unrestrained between the point loads, the design resistance \( (M_{sEd}) \) of the section will be reduced by lateral torsional buckling. The final section, allowing for the buckling resistance moment being less than the full resistance moment of the section, would have to be determined from experience. In this example, the bending strength \( (f_y) \) can be assumed to be about 240 N/mm², for preliminary sizing.

The plastic modulus required, \( W_{pf} = 605 \times 10^3/240 = 2520 \text{ cm}^3 \)

Two sections, both of the same weight per metre, have the required plastic modulus. They are:

(a) \( 533 \times 210 \times 101 \text{ UB, } W_{pf} = 2620 \text{ cm}^3 \)

(b) \( 610 \times 229 \times 101 \text{ UB, } W_{pf} = 2880 \text{ cm}^3 \)

Section (b) is appropriate if there is plenty of headroom, because of the increased stiffness. It is assumed for this example that depth is limited, and a \( 533 \times 210 \times 101 \text{ UB will be tried.} \)

1.4.4 Design buckling resistance moment

The design buckling resistance moment of a laterally unrestrained beam is given by the following equation:

\[ M_{sEd} = \chi_{LT} \beta W_{pf} f_y/\gamma_M \]

in which \( \chi_{LT} \) is the reduction factor for lateral-torsional buckling, from Table 5.5.2, for the appropriate value of the non-dimensional slenderness \( \lambda_{LT} \), using curve a for a rolled section.
In this example, full lateral restraint is provided at the support and at the load points b and c. In general, all segments need to be checked, but in this case they are all of equal length. The central segment b-c is subject to uniform moment, which is the most severe condition, so only b-c is checked.

**Segment b-c**
The value of $\lambda_{LT}$ can be determined using Annex F.

For segment b-c it is assumed that the secondary beams at b and c provide the following conditions:
- restraint against lateral movement,
- restraint against rotation about the longitudinal axis, and
- freedom to rotate in plan.

ie $k = k_w = 1.0.$

The following formula for $\lambda_{LT}$ may be used:

$$\lambda_{LT} = \frac{L\lambda_{LT}}{(C_1)^{0.5}\left[1 + \left(\frac{L/\theta_{LT}}{25.66}\right)^{0.25}\right]}$$

where $L$ is the length between b and c
- $I_z$ is the second moment of area about the z-z axis $= 2690 \text{ cm}^4$
- $I_w$ is the warping constant $= 1.82 \text{ dm}^4$
- $W_{pjy}$ is the plastic modulus about the y-y axis $= 2620 \text{ cm}^3$
- $I_t$ is the torsion constant $= 102 \text{ cm}^4$
- $C_1$ is the correction factor for the effects of any change of moment along the length L.

Between the points b and c the moment is approximately constant, therefore $C_1$ may be taken as $1.0.$

$$a_{LT} = (I_w/I_z)^{0.5}$$

$$= (1.82 \times 10^9/102)^{0.5} = 133.6 \text{ cm}$$

$$i_{LT} = (I_z/I_w/W_{pjy})^{0.25}$$

$$= (2690 \times 1.82 \times 10^9/2620)^{0.25} = 5.17 \text{ cm}$$

**Note** As an alternative to these calculations, the values of $i_{LT}$ and $a_{LT}$ can be obtained from section tables 5.

$$\lambda_{LT} = \frac{250/5.17}{1 + \left(\frac{250/133.6}{25.66}\right)^{0.25}} = 46.8$$

The non-dimensional slenderness is given by:

$$\bar{\lambda}_{LT} = (\lambda_{LT}/\lambda_i) (\beta_w)^{0.5}$$

where $\lambda_i = 93.9 \varepsilon$

$= 93.9 \sqrt{(235/275)} = 86.8$

$\beta_w = 1$ for class 1 or class 2 cross-sections

$$\bar{\lambda}_{LT} = 46.8/86.8 = 0.54$$
For rolled I sections, buckling curve a should be used.

\[ \chi_{LT} = 0.911 \]

The design buckling resistance moment for segment b-c is:

\[ M_{b,Rd} = \chi_{LT} \beta_w W_{pf} f_y/\gamma_M \]
\[ = 0.9106 \times 1.0 \times 2620 \times 275/1.05/10^3 \]
\[ = 625 \text{ kNm} \]

This is greater than the design moment (601 kNm) between b and c

\[ \therefore \text{satisfactory.} \]

### 1.4.5 Shear on web

In all cases where there are point loads on members it is prudent to check for the effects of shear. The following check should be carried out:

Shear at point loads, \( V_{sd} = 242.5 - 17/7.5 \times 2.5 = 237 \text{ kN} \)

The design shear resistance for a rolled I section is:

\[ V_{p,Rd} = \frac{1.04 h t_w (f_y/3)}{\gamma_M} \]
\[ V_{p,Rd} = \frac{1.04 \times 536.7 \times 10.9 \times 275/\sqrt{3}}{1.05 \times 10^3} = 920 \text{ kN} \]

Inspection shows that \( V_{sd} < \frac{V_{p,Rd}}{2} \) so no reduction in moment resistance due to shear in the web is necessary.

* Bearing, buckling and crushing of the web

If the beam is supported on seating cleats, the checks for web bearing, buckling and crushing given in Section 1.3.6 must be made. To satisfy the assumptions made in the design, both flanges must be held in place laterally, relative to each other. If seating cleats are used then the top flange must be held laterally. There is no requirement to prevent the flanges from rotating in plan, as \( k \) has been taken as 1.0.

### 1.4.6 Deflection check

In this case self-weight deflection is small and may be neglected. The point-load deflection can be considered by calculating the deflection from unit loads and then multiplying by the applied loads. Note that the serviceability loads are used for deflection checks.

For two point loads on a beam the maximum deflection is given by:

\[ \delta = \frac{F_k a}{24 E I_y} (3 L^2 - 4 a^2) \]

where \( F_k \) is the value of one point load
\( L \) is the span
\( a \) is the distance from the support to the adjacent point load
For this beam the unit load deflection is:

\[ \delta = \frac{1 \times 2500 (3 \times 7500^2 - 4 \times 2500^2)}{24 \times 210,000 \times 61 \times 700 \times 10^4} = 1.156 \times 10^{-4} \text{ mm} \]

\[ \delta_2 \text{ for variable actions} = 1.156 \times 10^{-4} \times 94 \times 10^3 = 109 \text{ mm} \]

\[ \delta_1 \text{ for permanent actions} = 1.156 \times 10^{-4} \times 69 \times 10^3 = 8.0 \text{ mm} \]

\[ \frac{\text{18.9 mm}}{\delta_2} \]

The limits based on the span are the same as for the fully restrained beam in Section 1.3:

\[ \delta_{\text{max}} = 30 \text{ mm} \]

\[ \delta_2 = 21.4 \text{ mm} \]

Both are greater than the sum of the deflections, so the Eurocode recommendations are satisfied.

\[ : \text{ satisfactory.} \]

---

**1.4.7 Design procedure using the concise document (C-EC3)**

This example can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following specific checks in which a simpler procedure is used.

**Design buckling resistance moment**

The procedure in Eurocode 3 for determining the buckling resistance moment has been simplified by calculating the bending strength, \( f_w \), using the modified equivalent slenderness \( \lambda_{LT} \sqrt{\beta_w} \) and then \( M_{b,\text{rd}} \), using:

\[ M_{b,\text{rd}} = \beta_w f_w W_{p0}/\gamma_M \]

\[ \beta_w = 1.0 \text{ for class 1 and class 2 sections} \]

**Segment b–c**

\( k = 1.0 \) (supports and restraint points free to rotate on plan)

\[ (k/C_t)^{0.5} = 1.0 \]

From section tables:

\[ i_{LT} = 5.17 \text{ cm} \]

\[ a_{LT} = 134 \text{ cm} \]

\[ L = 250 \text{ cm} \]

\[ \lambda_{LT} = \frac{(k/C_t)^{0.5}}{1 + \left[ \frac{L/a_{LT}}{25.66} \right]^{0.25}} \]

\[ \lambda_{LT} = \frac{1 \times 250/5.17}{1 + \left[ \frac{(250/134)^2}{25.66} \right]^{0.25}} \]

\[ = 46.8 \]

\[ \lambda_{LT} \sqrt{\beta_w} = 46.8 \times \sqrt{1.0} = 46.8 \]

---

References
\[ f_b = 250 \text{ N/mm}^2 \]

\[ M_{b,rd} = \frac{1.0 \times 250 \times 2620}{1.05 \times 10^3} \]

\[ = 623.8 \text{ kNm} \]

This is greater than the design moment (601 kNm) between b and c.

\[ \therefore \text{satisfactory.} \]
1.5 Unrestrained beam (B3)

This example has been prepared to show the method of checking a beam which is unrestrained between supports but carries a uniformly distributed load on the top flange, for example a beam supporting a wall only.

1.5.1 Loading

Characteristic values
It is assumed that all the load is permanent.

Permanent action \( G_k = 14.5 \text{ kN/m} \)
(including self-weight)

Partial safety factors
Permanent action \( \gamma_{G,\text{sup}} = 1.35 \)  
NAD Table 1

1.5.2 Design values

\[
F_d = \gamma_{G,\text{sup}} G_k = 1.35 \times 14.5 = 19.6 \text{ kN/m}
\]

Design moment
Maximum moment occurs at mid-span.

\[
M_{\text{md}} = F_d \frac{L^2}{8} = \frac{19.6 \times 7.5^2}{8} = 138 \text{ kNm}
\]

Design shear
Maximum shear occurs at the support.

\[
V_{\text{sd}} = 19.6 \times 7.5/2 = 73.5 \text{ kN}
\]

It is necessary to use iteration to determine the section required. An approximate final size of member can be found from tables.

1.5.3 Design buckling resistance moment

Try a 457 × 191 × 67 UB

Checking the resistance of this section follows the basic method shown in Section 1.4, but because the loading is applied to the top flange it will have a destabilising effect. This means that in determining \( \lambda_{T,T} \) account must be taken of the terms which include \( z_g \). For a rolled I or H section:

\[
\lambda_{T,T} = \frac{k L i_{LT}}{(C_{T})^{0.5} \left[ \left( \frac{k}{k_w} \right)^2 + \left( \frac{k L i_{LT}}{h_i} \right)^2 + \left( \frac{2 C_{T} z_g}{h_i} \right)^2 \right]^{0.5} - \frac{2 C_{T} z_g}{h_i}}^{0.5}
\]

Equation F.29

where

- \( k \) is the effective length factor for rotational restraint in plan (1.0 in this case)
- \( L \) is the length of the member = 7500 mm
- \( l_{LT} \) is taken as \( (I_y I_z) W_{z,\text{sup}}^{-2} \) = 46.7 mm
- \( I_z \) is the second moment of area about the z-z axis
- \( i_w \) is the warping constant
$W_{p\delta}$ is the plastic modulus about the y-y axis

$C_i$ is a factor that varies with moment gradient and end conditions

$k_w$ is the corrective length factor for warping, taken as 1.0 unless special provision is made to prevent warping

$C_i$ is a factor for the destablising term which varies with the moment gradient and end conditions

$z_y$ is the vertical distance of the load above the shear centre, which is negative if the load is below the shear centre

$h$ is the distance between the shear centres of the flanges

**Note** The values of $i_{LT}$ or $I_z$, $I_w$ and $W_{p\delta}$ can be determined from section tables.

\[
\lambda_{LT} = \frac{7500/46.7}{1.132 \left[ \left( \frac{1}{1} \right)^2 + \frac{1}{20} \left( \frac{1 \times 7500/46.7}{453.6/12.7} \right)^2 + \left( \frac{2 \times 0.459 \times 226.8}{440.9} \right)^{0.5} \right]^{0.5} - \frac{2 \times 0.459 \times 226.8}{440.9}^{0.5}}
\]

\[
= 124
\]

\[
\overline{\lambda}_{LT} = \frac{124}{86.8} = 1.43
\]

From this, $\lambda_{LT} = 0.40$

Buckling resistance moment $M_{b,Rd} = \lambda_{LT} \beta W_{p\delta} f_y / \gamma_M$

\[
= 0.4 \times 1 \times 1470 \times 275 / 1.05 / 10^3 = 154 \text{ kNm}
\]

$154 \text{ kNm} > 138 \text{ kNm}$

\[\therefore \text{ satisfactory.}\]

The remainder of the checks given in Section 1.4 should be made for this beam, depending on the support conditions. Note that both flanges must be held in place laterally, at the supports, to meet the design assumptions.

### 1.5.4 Design procedure using the concise document (C-EC3)\textsuperscript{2}

For the particular case of beams with unrestrained compression flanges subjected to destablising loads, the procedure in C-EC3 for determining the buckling resistance moment is no different from that given above.
1.6 Plate girder (B4)

The transfer beam (B4) shown in Figure 8 is 17.5 m long and carries the load from two columns together with the load from six secondary beams (B1) at first-floor level. It can be shown by a simple calculation (as in C-EC3) that the spacing of the secondary beams is such that for a flange width of 700 mm the plate girder will not suffer from lateral torsional buckling. For the loading shown, design a stiffened plate girder in grade Fe 430 steel.

Figure 8  Transfer beam B4 (dimensions in mm)

1.6.1 Loading

The recommendations given in British Standard BS 6399: Part 1 are used to determine the load on the transfer beam. Columns at points b and c support the load from a roof and three floors (see Figure 1). Therefore the imposed loads carried by the columns can be reduced by 30%. An area reduction on the imposed load on the floor supported by the transfer beam may also be made. The area supported by this beam is approximately 130 m², giving a reduction of 13%.

Table 2 shows the variable and permanent actions carried by column C1 at the roof and each floor level.

Table 2  Loading for column C1 (kN)

<table>
<thead>
<tr>
<th>Beam</th>
<th>Variable actions</th>
<th>Sub-totals</th>
<th>Cumulative totals</th>
<th>Permanent actions</th>
<th>Sub-totals</th>
<th>Cumulative totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>rooftop</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>28.1</td>
<td></td>
<td>93.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14.1</td>
<td></td>
<td>47.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>14.1</td>
<td></td>
<td>47.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>56.3</td>
<td>188.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4th floor |                  |            |                   |                   |            |                   |
| 1        | 94.0             |            | 76.0              |                   |            |                   |
| 2        | 46.9             |            | 34.7              |                   |            |                   |
| 3        | 46.9             |            | 34.7              |                   |            |                   |
| 4        | 0                | 187.8      | 145.4             |                   |            | 333.4             |

244.1

3rd floor |                  |            |                   |                   |            |                   |
| 187.8    |                  |            | 145.4             |                   |            | 478.8             |

as 4th floor | 187.8 | 431.9 | 478.8 |

2nd floor |                  |            |                   |                   |            |                   |
| 187.8    |                  |            | 145.4             |                   |            | 624.2             |

as 4th floor | 187.8 | 619.7 | 624.2 |
Characteristic values
Columns a, b and c

Loads at roof level
from Table 2
Variable action  = 56.3 kN
Permanent action = 188 kN

Loads at floor level
from Table 2
Variable action  = 187.8 kN
Permanent action = 145.4 kN

Total point load at b and c
Variable action  = 56.3 + 3 \times 187.8 = 619 kN
Permanent action = 188 + 3 \times 145.4 = 624.2 kN

Secondary beams (B2)
The point loads are taken as the end reactions from beams B1 (see Section 1.3).
Variable action  = 94 kN
Permanent action = 69 kN

Partial safety factors
Variable action  = 1.5
Permanent action = 1.35

NAD Table 1
1.6.2 Design values

The variable action carried by the columns at points b and c can be reduced by 30%.

\[ F_d = 1.35 \times 624.2 + 1.5 \times 0.7 \times 619.8 = 1493.5 \text{ kN} \]

Secondary beams

Variable action carried by the secondary beams can be reduced by 13%.

\[ F_d = 1.35 \times 69 + 1.5 \times 0.87 \times 94 = 216 \text{ kN} \]

Figure 9 shows the design values for actions, shear forces and bending moments.
1.6.3 Moment resistance of the section ignoring the web

For the interaction of moment and shear, three different approaches are available.

- As a simplification, Eurocode 3 permits the designer to assume that all the moment is resisted by the flanges alone and the web is checked for shear only.
- The moment is resisted by the full cross-section, and the web is designed for the resulting longitudinal stresses combined with shear. The design equations are given in clause 5.6.7.2 of Eurocode 3: Part 1.1.
- Part of the moment is resisted by the full cross-section and the remainder by the flanges alone.

The simplified (first) method will be used in this example.

\[ M_{c,Re} = A_t f_y / \gamma_M \]

Using this expression and assuming that the web is 2 m deep and the flange is 40 mm thick, the flange area required is:

\[
A_t = \frac{M_{c,Re} \gamma_M}{f_y} 
= \frac{14.752 \times 10^6 \times 1.05}{2040 \times 275} 
= 27611 \text{ mm}^2
\]

Flange width, \( b \) = 693 mm

\[ \therefore \text{ use } 700 \times 40 \text{ flange plates.} \]

Figure 10 shows the section of the plate girder

![Figure 10](image-url)

Figure 10 Section of plate girder (dimensions in mm)
1.6.4 Classification of the cross-section

Flange
The flanges are designed assuming that their plastic resistance will be reached. The flanges must, therefore, be at least class 2.

For \( t_f = 40 \text{ mm}, \ f_y = 275 \text{ N/mm}^2 \)

\[ \varepsilon = \left( \frac{235}{275} \right)^{0.5} = 0.924 \]

Outstand \( c = \frac{700}{2} = 350 \text{ mm} \)

\[ \frac{c}{t_f} = \frac{350}{40} = 8.75 \]

The class 2 limiting value \( c/t_f \) for the outstand of a welded section is 10 \( \varepsilon \),

\[ = 9.2 \]

\[ 8.75 < 9.2 \]

\[ \therefore \text{ class 2 flange.} \]

1.6.5 Web design

The determination of the web thickness has to be by experience, with a certain amount of trial and error. In this example a 13 mm plate is tried. This thickness is not common, but can be obtained from the mills and has been selected to illustrate design points associated with Eurocode 3.

\[ d/t_w = \frac{2000}{13.0} = 154 \]

As \( d/t_w > 69 \varepsilon \) the web must be checked for shear buckling.

Shear buckling resistance of web

Webs with intermediate stiffeners may be designed according to clause 5.6.3 or clause 5.6.4 of Eurocode 3: Part 1.1. The former method (the simple post-critical method) is used in this example.

Assume the stiffener spacing shown in Figure 11.

![Figure 11](stiffener_spacing.png)
The design shear buckling resistance is given by:

\[ V_{\text{ba, Rd}} = \frac{d t_w \tau_{w}}{{\lambda_w}^2/3} \]  

where 
- \( d \) is the depth of the web
- \( t_w \) is the thickness of the web
- \( \tau_{w} \) is the simple post-critical shear strength

\( \tau_{w} \) is based on the slenderness ratio, \( \overline{\lambda}_w \), of the web.

\[ \overline{\lambda}_w = \frac{37.4 \times \varepsilon \times k_i}{\sqrt{235 \times 0.5}} \]

\[ k_i = \frac{235}{275} = 0.85 \]

\( \varepsilon = \frac{235 \times 0.5}{275} = 0.924 \)

\( k_i \) is the buckling factor for shear.

In this example, \( a/d = 1.25 \), therefore (see Figure 10):

\[ k_i = 5.34 + \frac{4}{(a/d)^2} = 5.34 + \frac{4}{1.25^2} = 7.9 \]

\[ \overline{\lambda}_w = \frac{2000/13}{37.4 \times 0.924 \times \sqrt{7.9}} = 1.59 \]

As \( \overline{\lambda}_w \) is greater than 1.2:

\[ \tau_{w} = (0.9/\overline{\lambda}_w)(f_{w}/\sqrt{3}) \]

\[ \tau_{w} = (0.9/1.59)(275/\sqrt{3}) = 90 \text{ N/mm}^2 \]

\[ \therefore V_{\text{ba, Rd}} = \frac{2000 \times 13 \times 90}{1.05 \times 10^3} = 2228 \text{ kN} \]

This is greater than the applied shear (2212.5 kN),

\[ \therefore \text{satisfactory.} \]

### 1.6.6 Deflection check

For the case of two loads placed symmetrically on the span, the maximum deflection at the centre is given by:

\[ \delta = \frac{P a}{24 E I} (3 L^2 - 4 a^3) \]

In this case, the deflection may be obtained by using this formula three times for pairs of point loads. This gives the following expression:

\[ \delta = [F_1 \times 7500 (3 \times 17500^2 - 4 \times 7500^2) + F_2 \times 2500 (3 \times 17500^2 - 4 \times 2500^2) + F_3 \times 5000 (3 \times 17500^2 - 4 \times 5000^2)]/24 E I \]

\[ + F_{\text{wt}} \times 17500^3/(384 E I) \]
For permanent actions:

\[
\delta_1 = [693\,200 \times 7500 (3 \times 17\,500^2 - 4 \times 7500^2) \\
+ 69\,000 \times 2500 (3 \times 17\,500^2 - 4 \times 2500^2) \\
+ 69\,000 \times 5000 (3 \times 17\,500^2 - 4 \times 5000^2)] \\
/(24 \times 210\,000 \times 6\,694\,000 \times 10^4) + 5 \times 8.1 \\
\times 17\,500^3/(384 \times 210\,000 \times 6\,694\,000 \times 10^4)
\]

= 12.0 mm

For variable actions:

\[
\delta_2 = [499\,100 \times 7500 (3 \times 17\,500^2 - 4 \times 7500^2) \\
+ 81\,780 \times 2500 (3 \times 17\,500^2 - 4 \times 2500^2) \\
+ 81\,780 \times 5000 (3 \times 17\,500^2 - 4 \times 5000^2)] \\
/(24 \times 210\,000 \times 6\,694\,000 \times 10^4)
\]

= 9.2 mm

The limits given in Eurocode 3 for beams supporting columns are L/400 for \(\delta_{\text{max}}\) and L/500 for \(\delta_2\). Table 3 compares the calculated and limiting values.

**Table 3 Calculated and limiting values**

<table>
<thead>
<tr>
<th>Load</th>
<th>Calculated (mm)</th>
<th>Limit (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imposed (\delta_2)</td>
<td>9.2</td>
<td>35</td>
</tr>
<tr>
<td>Combined (\delta_{\text{max}})</td>
<td>21.2</td>
<td>44</td>
</tr>
</tbody>
</table>

From this table it can be seen that the deflections are well within the limits set by Eurocode 3.

### 1.6.7 Design of stiffeners at supports

This stiffener is detailed as a welded end-plate, and so need be checked only for buckling. The crushing check would be required if the plate girder were supported on a bracket.

**Crushing resistance**

This stiffener, together with a portion of the web, should be checked for crushing.

The design crushing resistance of the web is given by:

\[
R_{y,\text{Ra}} = (s_i + s_j) t_w f_{yw} / \gamma_{M1}
\]

where

- \(A_i\) is the area of the stiffener required
- \(s_i\) is the effective length of web
- \(s_j\) is the stiff bearing length (taken as zero for this example)

\[
s_j = 2 t_j (b/2t_w)^{0.5} (f_y b_y)^{0.5} (1 - (\sigma_{\text{Ed}}/f_y)^2)^{0.5}
\]

\[
b_y = 700 (\text{ie} < 25 \times 40 = 1000)
\]

\[
\sigma_{\text{Ed}} = 0 \text{ at the support}
\]
\[ s_y = 2 \times 40 \times (700/13)^{1.5} = 587.0 \text{ mm} \]

At the end of a member \( s_y \) should be halved

\[ \therefore s_y = 293.5 \text{ mm} \]

\[ \text{The crushing resistance of the stiffeners must be added.} \]
\[ \text{Crushing resistance of stiffeners} = A_s f_y / f_{yi} \]
\[ \text{where} \quad A_s \text{ is the area of the stiffener} \]

\[ \text{Design crushing action} = 2212.5 \text{ kN} \]

\[ \therefore 2212.5 \times 10^3 = A_s 275/1.05 + 293.5 \times 13 \times 275/1.05 \]

\[ \therefore A_s = 4632 \text{ mm}^2 \]

\[ \text{Try end-plate 425 mm} \times 20 \text{ mm} \]

\[ \text{Area} = 425 \times 20 = 8500 \text{ mm}^2 \]

**Check stiffener for buckling**

The effective section of the stiffener is shown in Figure 12. It satisfies the recommendations in Eurocode 3: Part 1.1.

![Figure 12](image)

**Dimensions and section properties**

Effective area, \( A_s \)
\[ = 425 \times 20 + 180 \times 13 \]
\[ = 10,840 \text{ mm}^2 \]

Second moment of area, \( I_s \)
\[ = \frac{20 \times 425^3}{12} + 180/12 \times 13^3 \]
\[ = 127.4 \times 10^6 \text{ mm}^4 \]

Radius of gyration, \( i_s \)
\[ = \sqrt{(I_s/A_s)} = \sqrt{(127.4 \times 10^6/10,840)} \]
\[ = 109 \text{ mm} \]

Allowing for weld, outstanding \( c \)
\[ = 190 \text{ mm} \]

\[ c/t_l = 206/20 = 10.3 \]

\[ \epsilon = \sqrt{(235/275)} = 0.924 \]

References

5.7.3 (3)

5.7.6 (1)

Figure 5.7.4

Table 5.3.1 (Sheet 3)
The class 3 limiting value $c/t_i$ for a welded outstand is

$14 \varepsilon = 12.94$

$10.3 < 12.94$

\[ \therefore \text{class 3 section.} \]

The design buckling resistance of the stiffener is:

\[ N_{b, Rd} = \chi \beta_A A \gamma_E / \gamma_M \]

where \[ \beta_A = 1.0 \text{ (the stiffener is class 3)} \]

\[ \gamma_M = 1.05 \]

\[ \chi \] is the reduction factor and is determined from Table 5.5.2 using buckling curve c

\[ \bar{\lambda} = \left( \lambda / \lambda_1 \right) \beta_A^{0.5} \]

\[ \lambda = 0.75 \frac{d}{t_i} = 0.75 \times 2000/109 = 13.76 \]

\[ \lambda_1 = 93.9 \varepsilon = 93.9 \times 0.924 = 86.8 \]

\[ \therefore \lambda / \lambda_1 = 13.76 / 86.8 = 0.159 \]

\[ \chi = 1.0 \]

\[ \therefore N_{b, Rd} = \frac{10840 \times 275}{1.05 \times 10^3} \]

\[ = 2839 \text{ kN} \]

$N_{b, Rd}$ is greater than the end reaction (2212.5 kN),

\[ \therefore \text{satisfactory.} \]

\[ \]

\[ \]

\[ \]

\[ \]

1.6.8 Intermediate stiffeners subject to external load

Intermediate stiffeners subject to externally applied loads should be checked for a stiffener force of:

\[ F_s = P + N_s \]

where \[ P \] is the externally applied load (216 kN)

\[ N_s \] is the compression force in the stiffener resulting from tension field action

\[ N_s = V_{sd} - d \frac{t_w \tau_{tb}}{\gamma_M} \]

where \[ V_{sd} \] is the design value of the shear force at the stiffener

\[ = 2197.5 \text{ kN} \]

\[ \tau_{tb} \] is the initial shear buckling strength

\[ A_s = \bar{\lambda} = 1.59 \text{ from previous calculations (see page 27)}, \]

\[ \tau_{tb} = \left( \frac{1}{\bar{\lambda}^{0.5}} \right) \left( f_{yw} / \sqrt{3} \right) \]

\[ \therefore \tau_{tb} = (1/1.59^{0.5}) (275/\sqrt{3}) = 62.8 \]
\[ N_s = 2197.5 - (2000 \times 13 \times 62.8/1.05/10^3) \]
\[ = 642 \text{ kN} \]

\[ F_s = 216 + 642 \]
\[ = 858 \text{ kN} \]

**Buckling resistance**

The effective section of the stiffener is shown in Figure 13 and satisfies the geometric recommendations in Eurocode 3.

![Figure 13](image)

**Dimensions and properties**

Effective area \( A \)
\[ = 2 \times 13 \times 100 + 2 \times 13 \times 180 \]
\[ = 7280 \text{ mm}^2 \]

Second moment of area \( I_y \)
\[ = \frac{13 \times 213^3}{12} + \frac{360 \times 13^3}{12} \]
\[ = 10.53 \times 10^6 \text{ mm}^4 \]

Radius of gyration \( i_y \)
\[ = \sqrt{I_y/A} = \sqrt{(10.53 \times 10^6/7280)} = 38 \text{ mm} \]

**Section classification**

Classify stiffener outstanding as a flange.

\[ c/t_1 = \frac{100}{13} = 7.69 \]

\[ \varepsilon = \sqrt{(235/275)} = 0.924 \]

Limit for class 3 section
\[ = 15 \varepsilon \]
\[ = 13.8 \]

\[ \therefore \text{ class 3 section.} \]

The design buckling resistance of a compression member is:

\[ N_{b,\text{RD}} = \chi \beta_A A f_y / \gamma_{	ext{MI}} \]

where \( \beta_A = 1.0 \) (the section is class 1)

\( \gamma_{	ext{MI}} = 1.05 \)

\( \chi \) is the reduction factor and is determined from Table 5.5.2 using buckling curve c

References

Table 5.3.1 (Sheet 3)

5.5.1.1 (1)

NAD Table 1

Table 5.5.2

31
\bar{\lambda} = (\lambda \lambda_1)^{0.65}
\lambda = 0.75 \times \delta_{ey} = 0.75 \times 2000/38 = 39
\lambda_1 = 93.9 \epsilon = 93.9 \times 0.924 = 86.7
\therefore \bar{\lambda} = 39/86.7 = 0.45

Using buckling curve c,
\chi = 0.87
\therefore N_{b, Rd} = \frac{0.87 \times 7280 \times 275}{1.05 \times 10^3} = 1659 \text{ kN}

N_{b, Rd} is greater than the stiffener design load (858 kN)
\therefore satisfactory.

Stiffener stiffness
The second moment of area of an intermediate transverse stiffener should satisfy the recommendations given in clause 5.6.5 (3).
\frac{a}{d} = \frac{2500}{2000} = 1.25 \text{ ie } \frac{a}{d} < \sqrt{2}
\therefore I_c \geq 1.5 d^3 t_w^3/\pi^2
= 1.5 \times 2000^3 \times 13^3/2500^2
= 4.22 \times 10^6 \text{ mm}^4

Second moment of area of stiffener is 10.53 \times 10^6 \text{ mm}^4.

10.53 \times 10^6 > 4.22 \times 10^6
\therefore satisfactory.

Flange induced buckling
To prevent the possibility of the flange buckling into the web, the web should satisfy the following requirements:
\frac{dt_w}{f_y} \leq k \left( \frac{A_w}{A_{te}} \right)^{0.65}
where
A_w \text{ is the area of the web}
A_{te} \text{ is the area of the compression flange}
k = 0.4 (flange is class 2)
\frac{dt_w}{210000}{275} \times \left( \frac{2000 \times \frac{13}{700 \times 40} \right)^{0.65}
= 294

Actual \frac{dt_w}{f_y} = 2000/13 = 154

Actual \frac{dt_w}{f_y} for the web is less than the limiting value for flange induced buckling.
\therefore satisfactory.
1.6.9 Integrity
Requirement A3 of the 1991 Building Regulations \[10\] must be satisfied. This states that buildings having five or more storeys shall be constructed so that in the event of an accident the building will not suffer collapse to an extent disproportionate to the cause.

Approved Document A \[11\] to the Building Regulations states that one way of meeting this requirement is to provide effective horizontal and vertical ties, in accordance with the recommendations given in paragraph 5.1a, Section 5, of the Approved Document.

That is the approach adopted in this example.

Figure 14 shows the final plate girder.

Two stiffeners are placed at b and c to coincide with the column flange

Figure 14 Details of plate girder (dimensions in mm)
1.6.10 Design procedure using the concise document (C-EC3)²

This example can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following specific checks in which a simpler procedure is used.

Moment resistance of the section

In C-EC3 the simplified method from the example is adopted. The method assumes that the applied moment is resisted by the flanges, and the shear is resisted by the web:

\[ M_{\text{Sl}} \leq M_{\text{c,Rd}} \text{ and } V_{\text{Sl}} \leq V_{\text{ba,Rd}} \]

\( M_{\text{c,Rd}} \) in C-EC3 is determined in the same way as in Eurocode 3, as already shown.

The design shear buckling resistance, \( V_{\text{ba,Rd}} \), is determined using the simple post-critical method. The tension field method is not addressed in C-EC3.

\[ V_{\text{ba,Rd}} = d t_w \tau_{\text{ba}} / \gamma_{M1} \]

\( t_w = 13 \text{ mm} \)

\( d = 2000 \text{ mm} \)

\( a/d = 1.25 \)

\[ d t_w = 153.8 \]

\[ \therefore \tau_{\text{ba}} = 90 \text{ N/mm}^2 \]

\[ \therefore V_{\text{ba,Rd}} = 2000 \times 13 \times 90 / 1.05 / 10^3 \]

\[ = 2228 \text{ kN} \]

\[ \therefore V_{\text{Sl}} > V_{\text{ba,Rd}} (2212.5 \text{ kN}) \]

\[ \therefore \text{satisfactory.} \]

Intermediate stiffener design

Intermediate stiffener subjected to an external load, \( P \), should be designed for a stiffener force of:

\[ F_s = P + N_s \]

where

\( N_s = V_{\text{Sl}} - d t_w \tau_{\text{re}} / \gamma_{M1} \text{ but } N_s \geq 0 \)

\( a/d = 1.25 \text{ and } d t_w = 153.8 \)

\[ \therefore \tau_{\text{re}} = 63 \text{ N/mm}^2 \]

\[ \therefore N_s = 2197.5 - 2000 \times 13 \times 63 / 1.05 / 10^3 \]

\[ = 637 \text{ kN} \]

\[ \therefore F_s = 216 + 637 \]

\[ = 853 \text{ kN} \]

The axial resistance of the stiffener is checked against this design force using the procedure in clause C-EC3 5.7.6.
1.7 Column (C1)

The internal column shown in Figure 15 is subject to loads from a roof and three floors. Design the column for the loading shown, in grade Fe 430 steel, as a member in simple framing using the requirements in the NAD.

Figure 15 Details of column C1 (dimensions in mm)
1.7.1 Loading

Table 4 shows the variable and permanent actions carried by the internal column C1. These are the same as in Table 2, and are repeated here for convenience.

Table 4  Loading for column C1 (kN)

<table>
<thead>
<tr>
<th>Beam</th>
<th>Variable actions</th>
<th>Sub-totals</th>
<th>Cumulative totals</th>
<th>Permanent actions</th>
<th>Sub-totals</th>
<th>Cumulative totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>28.1</td>
<td></td>
<td>93.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14.1</td>
<td></td>
<td>47.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>14.1</td>
<td>14.1</td>
<td>47.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>56.3</td>
<td>0</td>
<td>188.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th floor</td>
<td>1</td>
<td>94.0</td>
<td></td>
<td>76.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>46.9</td>
<td></td>
<td>34.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>46.9</td>
<td>46.9</td>
<td>34.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>187.8</td>
<td>0</td>
<td>145.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>244.1</td>
<td></td>
<td></td>
<td>333.4</td>
</tr>
<tr>
<td>3rd floor</td>
<td>187.8</td>
<td></td>
<td>145.4</td>
<td></td>
<td></td>
<td>478.8</td>
</tr>
<tr>
<td>as 4th floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd floor</td>
<td>187.8</td>
<td></td>
<td>145.4</td>
<td></td>
<td></td>
<td>624.2</td>
</tr>
<tr>
<td>as 4th floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Partial safety factors for loading

Permanent actions  \( \gamma_{G,\text{sup}} = 1.35 \)  
Variable actions  \( \gamma_{Q,\text{sup}} = 1.50 \)

1.7.2 Section properties

Consider column 1st floor to 2nd floor

The size of the column must be determined from experience and then checked for compliance with the Eurocode rules.

Try a 254 x 254 x 73 UC grade Fe 430

\[
\begin{align*}
&h = 254.0 \text{ mm} \\
&t_w = 8.6 \text{ mm} \\
&d/t_w = 23.3 \\
&A = 9290 \text{ mm}^2 \\
&I_w = 557 \times 10^6 \text{ mm}^4 \\
&I_t = 573 \times 10^4 \text{ mm}^4 \\
&W_{p/2} = 989 \times 10^3 \text{ mm}^3 \\
&W_{c/5} = 894 \times 10^3 \text{ mm}^3 \\
&b = 254.0 \text{ mm} \\
&t_t = 14.2 \text{ mm} \\
&c/t_t = 8.94 \\
&I_y = 114 \times 10^6 \text{ mm}^4 \\
&I_x = 38.7 \times 10^6 \text{ mm}^4 \\
&W_{p/2} = 989 \times 10^3 \text{ mm}^3 \\
&W_{c/5} = 894 \times 10^3 \text{ mm}^3 \\
&i_x = 64.6 \text{ mm} \\
&i_y = 111 \text{ mm} \\
&(L/L_i)^{0.25} = \left( \frac{1}{2} \cdot \frac{W_{p/2}^2}{W_{c/5}} \right)^{0.25} = \frac{38.7 \times 10^6 \times 557 \times 10^6}{(989 \times 10^3)^2} = 68.5 \text{ mm} \\
&(L/L_i)^{0.5} = \left( \frac{557 \times 10^6}{573 \times 10^3} \right)^{0.5} = 986 \text{ mm} \\
\end{align*}
\]

All the above properties, including \( i_{LT} \) and \( a_{LT} \), can be obtained from section property tables. 

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1.7.3 Classification of cross-section

This section is designed to withstand small moments in addition to axial force. Note that the section is always in compression.

For \( t_r = 14.2 \text{ mm} \)
\[ p_r = 275 \text{ N/mm}^2 \]
\[ \varepsilon = (235/p_r)^{0.5} = (235/275)^{0.5} = 0.924 \]  

**Flange** (subject to compression)

Class 1 limiting value of \( c/t_r \) for outstand of a rolled section is 10 \( \varepsilon \)

\[ 10 \varepsilon = 10 \times 0.924 = 9.24 \]
\[ c/t_r = 8.94 \]  

**Web** (subject to bending and compression)

Class 1 limiting value of \( d/t_w \) for web subject to bending and compression is 33 \( \varepsilon \).

\[ 33 \varepsilon = 33 \times 0.924 = 30.5 \]
\[ d/t_w = 23.3 \]

\[ c/t_r < 10 \varepsilon \text{ and } d/t_w < 33 \varepsilon \]

\[ \therefore \text{ class 1 section.} \]

1.7.4 Design value of actions

The recommendations given in British Standard BS 6399: Part 1 are used to determine the load on this column. The column supports the load from the roof and three floors, so the variable action can be reduced by 30%. From Table 4 the permanent and variable actions are:

\[ G_k = 624.2 \text{ kN} \]
\[ Q_k = 0.7 \times 619.7 = 433.8 \text{ kN} \]

Design value of axial compression \((N_{sd})\)

\[ = \Sigma y_{Gj} G_{kj} + y_{Qj} Q_{kj} \]
\[ = 1.35 \times 624.2 + 1.5 \times 433.8 \]
\[ = 1493 \text{ kN} \]

There is no need to consider pattern loading, but the effect of unbalanced loading on either side of the column must be taken into account.

Eccentricity to be taken as 100 mm from face of column.

\[ \therefore \text{ eccentricity (y-y axis)} \]
\[ = h/2 + 100 \text{ mm} = 254/2 + 100 \]
\[ = 227 \text{ mm} \]

\[ \therefore \text{ taking the beam reactions from Table 4, total design value of moment applied at level 2, } M_{y,sd}(\text{total}), \text{ is: } (1.35 \times 94 + 1.5 \times 76) \times 227/10^3 \]
\[ = 54.7 \text{ kNm} \]
Moment to be divided in proportion to the column stiffnesses. Column stiffnesses are equal above and below level 2, so the moment is divided equally,

\[ M_{sd} = \frac{54.7}{2} = 27.3 \text{ kNm} \]

This moment has no effect on the floors above or below, so the moment increases linearly from 0 at level 1 to 27.3 kNm at level 2, and from 0 at level 3 to 27.3 kNm at level 2 in the upper length. It should be noted that \( C_1 = 1.0 \) in the calculations.

### 1.7.5 Resistance of cross-section

For a class 1 section without bolt holes, the reduced design plastic resistance moment, allowing for the axial force, is:

\[ M_{Ny,Rd} = \frac{M_{p,f,Rd} (1 - n)}{1 - 0.5 a} \]

where

\[ n = \frac{N_{sd}}{N_{p,Rd}} \]

\[ a = \frac{(A - 2 b t_f)}{A} \text{ but } a \leq 0.5 \]

**To calculate \( M_{p,f,Rd} \)**

For bending about one axis, the design moment of resistance is:

\[ M_{p,f,Rd} = W_{p,f,f,s} / f_{m0} \]

\[ = 989 \times 10^3 \times 275 / 1.05 / 10^9 \]

\[ = 259 \text{ kNm} \]

**To calculate \( n \) and \( a \)**

Applied axial force, \( N_{sd} = 1493 \text{ kN} \)

For a member subject to axial compression, the design plastic resistance of the cross-section is:

\[ N_{p,Rd} = A f_y / f_{m0} \]

\[ = \frac{9290 \times 275}{1.05 \times 10^3} = 2433 \text{ kN} \]

\[ \therefore \ n = 1493 / 2433 = 0.61 \]

\[ a = \frac{A - 2 b t_f}{A} = \frac{9290 - 2 \times 254 \times 14.2}{9290} = 0.22 \]

\[ M_{Ny,Rd} = M_{p,f,Rd} (1 - n)(1 - 0.5 a) \]

\[ = 259 (1 - 0.61)(1 - 0.5 \times 0.22) \]

\[ = 113.5 \text{ kNm} \]

\[ \text{ie} > M_{sd} (27.3 \text{ kNm}), \]

\[ \therefore \text{ satisfactory}. \]
1.7.6 Buckling resistance of the member
A class 1 member subject to combined bending and axial compression should be checked for the following modes of failure:
- Flexural buckling (clause 5.5.4 (1)), and
- Lateral torsional buckling (clause 5.5.4 (2)).

**Flexural buckling**
A class 1 member subject to moment about the major axis only, should satisfy the following:
\[
\frac{N_{sl}}{\chi_{min} A f_y / \gamma_{Mf}} + \frac{k_y M_{x,sl}}{W_{p,ef} f_y / \gamma_{Mf}} \leq 1.0
\]

Applied axial force, \( N_{sl} = 1493 \text{ kN} \)

To calculate \( \chi_{min} \)
\( \chi_{min} \) is the lesser of \( \chi_y \) and \( \chi_z \), where \( \chi_y \) and \( \chi_z \) are the reduction factors from clause 5.5.1 for the y-y and z-z axes respectively.

**Determination of \( \chi_y \)**
The reduction factor \( \chi_y \) depends on the slenderness about the y-y axis.

Assuming that the connections between the column and the primary members at levels 1 and 2 are effectively pinned, then the slenderness about the y-y axis is:
\[
\begin{align*}
\lambda_y &= \frac{f_y}{\chi_y} = 4000/111 = 36.0 \\
\lambda_1 &= \frac{\pi (E/\chi_y)}{\chi_y} = 93.9 \varepsilon = 86.8 \\
\bar{\chi}_y &= \lambda_y \lambda_1 \beta_A \sqrt{0.86} = 36/86.8 \times 1.05 = 0.41
\end{align*}
\]

From Table 5.5.3, use buckling curve b
\( \chi_y = 0.922 \)

**Determination of \( \chi_z \)**
\( \ell = L = 4000 \text{ mm} \)
\( \lambda_z = \frac{f_y}{\chi_z} = 4000/64.6 = 61.9 \)
\( \bar{\chi}_z = \lambda_z \lambda_1 \beta_A \sqrt{0.86} = 61.9/86.8 \times 1.05 = 0.71 \)

From Table 5.5.3, use curve c \( \chi_z = 0.718 \)

\( \therefore \chi_{min} = \chi_z = 0.718 \)

To calculate \( k_y \)
\[
k_y = 1 - \frac{\chi_y N_{sl}}{A f_y} \quad \text{but} \quad k_y \leq 1.5
\]

where \( \mu_y = \frac{\bar{\chi}_y (2 \beta_{My} - 4) + \frac{W_{p,ef}}{W_{ef}}}{2} \quad \text{but} \quad \mu_y \leq 0.90 \)
The NAD assumes a uniform moment on the column (\( \psi = 1.0 \)), therefore \( \beta_{My} \) (the equivalent uniform moment factor) is taken as 1.1 for bending about both axes.

\[ \beta_{My} = 1.1 \]

\[ \mu_y = 0.41 (2 \times 1.1 - 4) + \frac{989 \times 10^3 - 894 \times 10^3}{894 \times 10^3} = -0.63 \]

\[ k_y = 1 - \frac{(-0.63 \times 1493 \times 10^3)}{0.922 \times 9290 \times 275} \]

\[ k_y = 1.4 (< 1.50) \]

\[ \frac{N_{sd}}{\chi_{mn} A f_y / \gamma_{MI}} + \frac{k_y M_{y, sd}}{W_{p,y} f_y / \gamma_{MI}} \leq 1.0 \]

\[ \frac{1493 \times 10^3}{0.718 \times 9290 \times 275/1.05} + \frac{1.4 \times 27.3 \times 10^6}{989 \times 10^3 \times 275/1.05} = 1.000 \]

\[ \therefore \text{satisfactory for flexural buckling.} \]

---

**Lateral torsional buckling**

A class 1 section should satisfy the following:

\[ \frac{N_{sd}}{\chi_e A f_y / \gamma_{MI}} + \frac{k_{LT} M_{y, sd}}{\chi_{LT} W_{p,y} f_y / \gamma_{MI}} \leq 1.0 \]

Applied axial force \( N_{sd} = 1493 \) kN

Applied moment \( M_{y, sd} = 27.3 \) kNm

To calculate \( k_{LT} \)

\[ k_{LT} = 1 - \frac{\mu_{LT} N_{sd}}{\chi_e A f_y} \quad \text{but} \quad k_{LT} \leq 1 \]

where \( \mu_{LT} = 0.15 \bar{x}_y \beta_{MLT} - 0.15 \quad \text{but} \quad \mu_{LT} \leq 0.9 \)

\[ \beta_{MLT} = 1.1 \]

\[ \therefore \mu_{LT} = 0.15 \times 0.71 \times 1.1 - 0.15 = -0.0329 \]

\[ k_{LT} = 1 - \frac{(-0.0329) \times 1493 \times 10^3}{0.718 \times 9290 \times 275} = 1.027 > 1.0 \quad \therefore k_{LT} = 1.0 \]

---

5.5.4 (2)
To calculate $\xi_{LT}$

The value of $\xi_{LT}$ can be determined from Table 5.5.2 for the appropriate value of the non-dimensional slenderness $\lambda_{LT}$.

$$\lambda_{LT} = \lambda_{LT} \beta_1 \beta_w 0.5$$

$\lambda_{LT}$ should be determined from Annex F of Eurocode 3: Part 1.1, with a $C_t$ factor taken as 1.0.

For a nominally pin-ended I or H section with end-moment loading only, the value of $\lambda_{LT}$ can be obtained from:

$$\lambda_{LT} = \frac{L/h_T}{C_1^{0.5} \left[ 1 + \frac{(L/h_T)^2}{25.66} \right]^{0.25}}$$

$$= \frac{4000/68.5}{1.0^{0.5} \left[ 1 + \frac{(4000/986)^2}{25.66} \right]^{0.25}} = 51.6$$

$$\xi_{LT} = \lambda_{LT} \beta_1 \beta_w 0.5 = 51.6/86.8 = 0.59$$

∴ from Table 5.5.2, using curve a (for rolled sections) \(5.5.2\) (4)

$$\eta_{LT} = 0.893$$

$$\frac{N_{sd}}{\chi_{LT} A f_y/\gamma_M} + \frac{k_{LT} M_{x,sl}}{\chi_{LT} W_{pr} f_y/\gamma_M} \leq 1.0$$

$$= \frac{1493 \times 10^3}{0.718 \times 9290 \times 275/1.05} + \frac{1.0 \times 27.3 \times 10^6}{0.893 \times 989 \times 10^3 \times 275/1.05} = 0.98$$

$$0.98 \leq 1.0$$

∴ satisfactory for lateral torsional buckling.

### 1.7.7 Design procedure using the concise document (C-EC3)

This example can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following specific checks in which a simpler procedure is used.

**Resistance of the cross-section**

C-EC3 presents a simplified equation for the reduced moment of resistance in the presence of axial force.

For class 1 and class 2 sections with low shear:

$$M_{sd} \leq M_{ny,rd}$$

where $M_{ny,rd} = 1.11 M_{p_y}$ (1 - n) but $M_{ny,rd} \leq M_{p_y}$

$$n = \frac{N_{sd}/N_{p_y,rd}}{1493/2433} = 0.61$$

$$M_{p_y,rd} = 259.0 \text{ kNm}$$

∴ $M_{ny,rd} = 1.11 \times 259.0 (1 - 0.61) = 112.1 \text{ kNm} (< M_{p_y})$

ie $> M_{sd}$ (27.3 kNm)

∴ satisfactory.
Buckling resistance of the member

The procedure is similar to that given in the Eurocode, except for the following checks in which a simpler procedure is used.

Major axis buckling mode:

\[
\frac{N_{sd}}{N_{b.y, Rd}} + \frac{k_y M_{s,y, sd}}{\eta M_{c,y, Rd}} \leq 1.0
\]

and

\[
\frac{N_{sd}}{N_{b,z, Rd}} \leq 1.0
\]

\[
N_{b.y, Rd} = \beta_A \ell_A A/\gamma_{Mi}
\]

\[
\beta_A = 1
\]

\[
A = 9290 \text{ mm}^2
\]

\[
\gamma_{Mi} = 1.05
\]

\[
\lambda_y = \frac{f}{i_y} = \frac{4000}{111} = 36.0
\]

\[
\lambda_y \sqrt{\beta_A} = 36 \sqrt{1} = 36.0
\]

Using buckling curve b:

\[
f_c = 253 \text{ N/mm}^2
\]

\[
N_{b.y, Rd} = 1.0 \times 253 \times 9290/1.05 \times 10^3 = 2238 \text{ kN}
\]

\[
M_{c,y, Rd} = M_{p,y, Rd}
\]

\[
= 259.0 \text{ kNm}
\]

\[
k_y = 1.5 \text{ (conservative)}
\]

Note C-EC3 also contains a procedure for determining a more representative (less conservative) value of k.

\[
\eta = \frac{\gamma_{Mi}}{\gamma_{Mi}} = \frac{1.05}{1.05} = 1.0
\]

\[
\frac{N_{sd}}{N_{b.y, Rd}} + \frac{k_y M_{s,y, sd}}{\eta M_{c,y, Rd}} = \frac{1493}{2238} + \frac{1.5 \times 27.6}{1.0 \times 259} = 0.83
\]

ie < 1.0

\[
N_{b,z, Rd} = \beta_A \ell_A A/\gamma_{Mi}
\]

\[
\beta_A = 1
\]

\[
A = 9290 \text{ mm}^2
\]

\[
\gamma_{Mi} = 1.05
\]

\[
\lambda_z = \frac{f}{i_z} = \frac{4000}{64.6} = 61.9
\]

\[
\lambda_z \sqrt{\beta_A} = 61.9 \sqrt{1} = 61.9
\]
Using buckling curve c:

\[ f_c = 197 \text{ N/mm}^2 \]

\[ \therefore \frac{N_{b,z,Rd}}{N_{b,z,Rd}} = \frac{1.0 \times 197 \times 9290}{1.05 \times 10^3} = 1743 \text{ kN} \]

\[ \frac{N_{sl}}{N_{b,z,Rd}} = \frac{1493}{1743} = 0.86 \]

ie < 1.0

\[ \therefore \text{ satisfactory.} \]

**Lateral torsional buckling mode**

\[ \frac{N_{sl}}{N_{b,z,Rd}} + \frac{k_{LT} M_{y,sl}}{M_{b,Rd}} \leq 1.0 \]

\[ N_{b,z,Rd} = \beta_A \frac{f_c}{\gamma_M} \]

\[ \lambda_z = \frac{f_c}{\beta_A} \frac{A_y}{\gamma_M} = 4000/64.6 = 61.9 \]

Using buckling curve c:

\[ f_c = 197 \text{ N/mm}^2 \]

\[ \therefore N_{b,z,Rd} = \frac{1.0 \times 197 \times 9290}{1.05 \times 10^3} = 1743 \text{ kN} \]

\[ M_{b,Rd} = \beta_W f_c W_{pcy}/\gamma_M \]

\[ \lambda_{LT} = \left( \frac{k}{C_1} \right)^{0.5} \times \left[ 1 + \frac{L/\lambda_{LT}}{(L/\lambda_{LT})^2 + 0.25} \right] \]

\[ k = 1.0 \text{ and } C_1 = 1.0 \text{ (simple construction)} \]

\[ \therefore \lambda_{LT} = 1.0 \times \left[ \frac{4000/68.5}{1 + \left( \frac{4000/986)^2}{25.66} \right)^{0.25} \right] = 51.6 \]

\[ \therefore f_b = 245 \text{ N/mm}^2 \]

\[ \therefore M_{b,Rd} = \frac{1.0 \times 245 \times 989}{1.05 \times 10^3} = 230.8 \text{ kNm} \]

\[ k_{LT} = 1.0 \text{ (conservative value)} \]

\[ \therefore \frac{N_{sl}}{N_{b,z,Rd}} + \frac{k_{LT} M_{y,sl}}{M_{b,Rd}} = \frac{1493}{1743} + \frac{1.0 \times 27.6}{230.8} = 0.98 \]

ie < 1.0

\[ \therefore \text{ satisfactory.} \]
1.8 General requirements for structural integrity

1.8.1 Tying forces
Beams and their connections should be designed to resist tie forces, so that they will limit the progressive spread of damage in the event of an accident. These forces are defined as:

\[\begin{align*}
0.5 w \cdot s \cdot L_a & \text{ for internal beams, and} \\
0.25 w \cdot s \cdot L_a & \text{ for edge beams,}
\end{align*}\]

where \( w \) is the total design permanent and variable load \\
\( s \) is the mean spacing of the ties \\
\( L_a \) is normally the span of the beam (See the NAD for a full definition).

Also, the tie forces should be not less than 75 kN for floors or 40 kN for roofs.

For this example the terms have the following values:

\[w_t = \gamma_0 G_k + \gamma_0 Q_k\]

For the floors \[= 1.35 \times 3.7 + 1.5 \times 5.0 = 12.5 \text{ kN/m}^2\]

For the roof \[= 1.35 \times 5.0 + 1.5 \times 1.5 = 9.0 \text{ kN/m}^2\]

\(s\) for floors and roof:

For edge beams \[2.5 \text{ m}\]
For secondary beams \[2.5 \text{ m}\]
For main beams \[7.5 \text{ m}\]

\(L_a\) for all beams in floor and roof \[= 7.5 \text{ m}\]

Column tie forces
The columns must be restrained at the periphery of the structure. The restraints must be capable of resisting a tie force of not less than 1% of the axial force in the column.

The maximum forces in the columns may be determined as follows:

Area supported by external column \[= 7.5 \times 3.75 = 28.1 \text{ m}^2\]

Column load at roof level
\[w_t \times \text{area} = 9 \times 28.1 = 253 \text{ kN}\]

The restraint tie force \[= 1\% = 2.53 \text{ kN}\]

Experience will show that normally this is not critical.

Column load at 1st floor

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>253 kN</td>
</tr>
<tr>
<td>2nd to 4th floors</td>
<td>1054 kN</td>
</tr>
<tr>
<td>Plate girder reaction</td>
<td>2213 kN</td>
</tr>
<tr>
<td>Edge beams</td>
<td>105 kN</td>
</tr>
<tr>
<td>Cladding</td>
<td>960 kN</td>
</tr>
<tr>
<td>Total</td>
<td>4585 kN</td>
</tr>
</tbody>
</table>

The restraint tie force = 1% of the total axial force \[= 46 \text{ kN}\]
Roof tie forces

Edge beams  \(0.25 \times 9.0 \times 2.5 \times 7.5\) = 42 kN
Secondary beams  \(0.5 \times 9.0 \times 2.5 \times 7.5\) = 84 kN
Main beams  \(0.5 \times 9.0 \times 7.5 \times 7.5\) = 253 kN

These are all greater than 40 kN and 1% of the column load (2.53 kN), and should be used in the design of the members and connections.

Floor tie forces

Edge beams  \(0.25 \times 12.5 \times 2.5 \times 7.5\) = 59 kN
Secondary beams  \(0.5 \times 12.5 \times 2.5 \times 7.5\) = 117 kN
Main beams  \(0.5 \times 12.5 \times 7.5 \times 7.5\) = 352 kN

The edge beams and their connections should be designed for 75 kN, which is greater than both the calculated tie force and 1% of the column load (46 kN). The main and secondary beams, along with their connections, should be designed to resist the forces given above.

1.8.2 Frame imperfections

Frame imperfections are allowed for by assuming an equivalent system of internal forces, including equivalent horizontal forces applied to those systems which provide the lateral stability of the frame. The reactions generated by these equivalent forces are balanced by the equal and opposite reactions of the closed system of internal forces, and so are not added to the total shear force at the foundations due to external actions. The effect on any individual foundation can easily be found by considering the imperfection angle \(\phi\).

To determine the horizontal actions to be resisted by the bracing system, the total design load at each floor is required. This is then used to determine the equivalent horizontal forces applied at each level.

From previous calculations the total unit design load intensities are:

- Roof 9.0 kN/m²
- Floors 12.5 kN/m²

In addition, an allowance should be made on the design load for the weight of the columns and beam casings; say 0.3 kN/m² for the roof and floors 2 to 4, and 1.0 kN/m² for the 1st floor, including the plate girder and its casing.

The cladding has a characteristic load of 0.8 kN/m² and a design load of 0.89 \(\times 1.35\) = 1.2 kN/m².

The plan size of the building is 17.5 m \(\times\) 52.5 m, giving an area of 919 m².

The perimeter of the building is \(2 \times (17.5 + 52.5)\) = 140 m.

The weight of the cladding is, therefore, \(4.8 \times 140\) = 672 kN/floor.
The floor loads are given in Table 5.

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Total design loads on each floor (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Roof</td>
</tr>
<tr>
<td></td>
<td>9.3 × 919</td>
</tr>
</tbody>
</table>

To obtain the equivalent horizontal forces for checking the frame, the total floor load is multiplied by the imperfection angle \( \phi \) which is obtained from:

\[
\phi = k_c k_s \phi_o
\]

where

\[
k_c = (0.5 + 1/n_s)^{0.5} \quad \text{but} \quad k_c \leq 1.0
\]

\[
\phi_o = 1/200
\]

\[
k_s = (0.2 + 1/n_s)^{0.5} \quad \text{but} \quad k_s \leq 1.0
\]

\( n_s \) is the number of columns supporting all floors and carrying more than 50% of the average column load in the plane under consideration.

\( n_s \) is the number of storeys

In this example the value of \( n_s \) for the transverse imperfections = 2,

\[
\therefore \quad k_c = (0.5 + 1/2)^{0.5} = 1
\]

The value for the longitudinal imperfections is based on there being eight columns in one plane, giving:

\[
k_c = (0.5 + 1/8)^{0.5} = 0.791
\]

The value of \( n_s \) in both directions is the same, 5. This gives a value of:

\[
k_s = (0.2 + 1/5)^{0.5} = 0.634
\]

The final imperfections will be:

- Transverse \( 1 \times 0.634 \times 1/200 = 1/315 \)
- Longitudinal \( 0.791 \times 0.634 \times 1/200 = 1/399 \)

Table 6 gives the equivalent horizontal forces which should be applied to the frame.

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Equivalent horizontal forces at each floor (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design load</td>
</tr>
<tr>
<td></td>
<td>Roof</td>
</tr>
<tr>
<td>8547</td>
<td>27.1</td>
</tr>
</tbody>
</table>
These forces will be distributed equally between each bracing system. As the structure is more than four storeys high, a minimum of two independent systems of bracing is required in each direction (see plan, Figure 16). In this example, a single cross-braced bay has been incorporated in each of the two end-walls, and two braced bays have been built into the rear elevation.

![Diagram of building bracing system]

**Figure 16** Plans of building showing bracing

As the longitudinal bracing is provided on one elevation only, there will be a couple induced, and this will have to be resisted by the transverse bracing. In calculating this couple, it is assumed that the wind acts in the longitudinal direction.

To ensure that the structure remains square in plan and transmits the forces to the vertical bracing systems, plan bracing will be required. This will normally be provided by the floors acting as diaphragms but, if necessary, may take the form of individual bracing members incorporated in the floors.

### 1.8.3 Wind loading on bracing systems

The wind loading in these examples is taken from British Standard CP 3: Chapter V: Part 2, where \( q_p = 0.68 \text{ kN/m}^2 \) and \( C_r = 1.2 \text{ kN/m}^2 \).

The force on each of the transverse braced bays

\[
q_p C_r \text{ length/2} = 0.68 \times 1.2 \times 52.5/2 = 21.4 \text{ kN/m}
\]

The total force to be resisted by the longitudinal bracing

\[
17.5 \times 0.75 \times 0.68 = 8.9 \text{ kN/m}
\]

The force in the transverse bracing resulting from wind acting on the end-bays is determined by multiplying the total force on the longitudinal bracing by the ratio of half the width of the building by its length,

\[
\text{ie multiplying factor} = \frac{17.5}{2 \times 52.5} = 0.167
\]

It is assumed that the roof will resist the wind from half a storey height (2 m); the floors, except the first, will resist the wind from 4 m, and the 1st floor will resist the wind from 4.5 m.
Figure 17 shows the distribution of wind between floors.

**Summary of bracing loads**

Unlike current UK practice, the forces on bracing resulting from imperfections and wind loads are additive.

Tables 7 and 8 summarise the design actions on both the transverse and the longitudinal bracing.

**Table 7  Design actions on transverse bracing**

<table>
<thead>
<tr>
<th></th>
<th>Wind on front elevation</th>
<th>Resultant couple from wind on ends</th>
<th>Transverse imperfections</th>
<th>Longitudinal imperfections</th>
<th>Design actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>42.8</td>
<td>3.0</td>
<td>13.6</td>
<td>3.6</td>
<td>63.0</td>
</tr>
<tr>
<td>4th floor</td>
<td>85.6</td>
<td>5.9</td>
<td>18.6</td>
<td>5.2</td>
<td>115.3</td>
</tr>
<tr>
<td>3rd floor</td>
<td>85.6</td>
<td>5.9</td>
<td>18.6</td>
<td>5.2</td>
<td>115.3</td>
</tr>
<tr>
<td>2nd floor</td>
<td>85.6</td>
<td>5.9</td>
<td>18.6</td>
<td>5.2</td>
<td>115.3</td>
</tr>
<tr>
<td>1st floor</td>
<td>96.3</td>
<td>6.7</td>
<td>20.7</td>
<td>5.4</td>
<td>129.1</td>
</tr>
</tbody>
</table>

**Table 8  Design actions on longitudinal bracing**

<table>
<thead>
<tr>
<th>Imperfections</th>
<th>Wind</th>
<th>Design actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>21.4</td>
<td>39.2</td>
</tr>
<tr>
<td>4th floor</td>
<td>31.0</td>
<td>66.6</td>
</tr>
<tr>
<td>3rd floor</td>
<td>31.0</td>
<td>66.6</td>
</tr>
<tr>
<td>2nd floor</td>
<td>31.0</td>
<td>66.6</td>
</tr>
<tr>
<td>1st floor</td>
<td>32.6</td>
<td>76.7</td>
</tr>
</tbody>
</table>
1.8.4 Design procedure using the concise document (C-EC3)

Frame imperfections

C-EC3 contains a summary table of imperfection factors, $\phi$, for different numbers of columns, $n_c$, and storeys, $n_s$, thereby simplifying the calculation:

Number of storeys, $n_s = 5$

Number of columns, $n_c$

- Transverse = 2 $\therefore \phi = 1/315$
- Longitudinal = 8 $\therefore \phi = 1/400$

References

C-EC3 5.2.4.3
C-EC3 Table 5.3
C-EC3 Table 5.3
1.9 Beam-to-beam connection (B1–B2)

1.9.1 Initial design information
Design the connection between a pair of secondary beams in Section 1.3 (406 x 140 x 46 UB, member reference B1), and the primary beam in Section 1.4 (533 x 210 x 101 UB, member reference B2).

From Section 1.3, for a secondary beam the design ultimate reaction is:

\[ V_{ud} = 117 \text{ kN} \]

The connection is to be nominally pinned so that deformations of the secondary beam can occur without inducing significant moments in the primary beams.

The connection will be designated as a ‘Category A: Bearing type’ shear connection using a partial depth flexible end-plate. While the flexible end-plate is a commonly used connection type, it can result in increased erection effort, particularly when two are connected back-to-back through a beam or column web. This should be borne in mind when selecting an appropriate connection detail.

1.9.2 Basic details
To achieve the required flexibility, the connection will be detailed in accordance with Volume 1 of Joints in simple construction\(^2\) (see Figure 18).

![Diagram of beam-to-beam connection](image)

**Figure 18** Beam-to-beam connection details (dimensions in mm)

**Note** The proposed end-plate is 200 mm wide x 250 mm deep x 8 mm thick. The dimensions of the plate have been selected to comply with the recommended minimum bolt gauge of 140 mm and a plate depth greater than 0.6 of the depth of the supported beam.

It is envisaged that a four-bolt connection will be adequate. Using an end distance of 35 mm, the bolt pitch is 180 mm.

Minimum end distance, \( e_1 \quad = \quad 1.2 \, d_n \quad = \quad 1.2 \times 22 \)
\[ = \quad 26.4 \text{ mm, } \quad ie < 35 \text{ mm} \]

Normal edge distance, \( e_2 \quad = \quad 1.5 \, d_n \quad = \quad 1.5 \times 22 \)
\[ = \quad 33 \text{ mm, } \quad ie > 30 \text{ mm} \]
Although the edge distance is less than normal, it is acceptable provided that the bearing resistance is reduced accordingly and that the edge distance is not less than the minimum value of 1.2 \( d_e \) (26.4 mm).

Assumed notch depth, \( d_e = 32 \text{ mm} \)
Assumed notch length, \( c = 110 \text{ mm} \)

1.9.3 Shear resistance of the bolt group
Treating the force on each shear plane separately:
Shear per bolt = \( 117/4 = 29.3 \text{ kN} \)

Shear resistance of bolt, \( F_{v,Rd} = \frac{0.6 f_{ub} A_s}{\gamma_{Mh}} \)

where \( \gamma_{Mh} \) is the material factor = 1.35
\( f_{ub} \) is the ultimate tensile strength of the bolt = 800 N/mm\(^2\)
\( A_s \) is the tensile stress area of the bolt = 245 mm\(^2\)

\[ F_{v,Rd} = \frac{0.6 \times 800 \times 245}{1.35 \times 10^3} \]
\[ = 87.1 \text{ kN/} \text{plane} \]

\[ i_e > 29.3 \text{ kN}, \]

\[ : \text{satisfactory.} \]

1.9.4 Check shear resistance of the end-plate
The reduction in shear resistance resulting from the presence of the fasteners can be ignored if:

\[ \frac{A_{v,net}}{A_v} \geq \frac{f_v}{f_u} \]
\[ A_{v,net}/A_v = \frac{(2 \times 250 \times 8) - (4 \times 8 \times 22)}{2 \times 250 \times 8} \]
\[ = 0.824 \]
\[ f_v/f_u = 275/430 \]
\[ = 0.640 \]

\[ i_e \leq 0.824 \]

\[ : \text{the presence of the holes can be ignored and the gross area of the plate can be used.} \]

Design plastic shear resistance

\[ V_{pf,Rd} = \frac{A_v f_y}{\sqrt{3} \times \gamma_{M0}} \]

where \( f_y = 275 \text{ N/mm}^2 \)
\( \gamma_{M0} = 1.05 \)
\( A_v = 2 \times 8 \times 250 = 4 \times 10^3 \text{ mm}^2 \)

\[ \text{References} \]
6.5.5 (10)
6.5.1.3 (2)

NAD Table 1
3.3.2.1 (3)
\[
V_{\text{frD}} = \frac{4 \times 10^3 \times 275}{\sqrt{3} \times 1.05 \times 10^3} = 604.8 \text{ kN}
\]

ie \( V_{\text{frD}} > 117 \text{ kN} \), \( \therefore \) the plates are adequate in shear,

\( \therefore \) **satisfactory.**

### 1.9.5 Design end-plate weld

The end-plate is connected to the beam web by two full depth fillet welds.

Fillet weld shear strength, \( f_{w,d} = \frac{f_u}{\beta_w \gamma_{MW} \sqrt{3}} \)

where \( f_u = 430 \text{ N/mm}^2 \)

\( \beta_w \) is a correlation factor for the grade of steel being used. For Fe 430:

\( \beta_w = 0.85 \)

\( \gamma_{MW} = 1.35 \)

\( \therefore f_{w,d} = \frac{430}{0.85 \times 1.35 \times \sqrt{3}} = 216 \text{ N/mm}^2 \)

Total length of weld = \( 250 \times 2 = 500 \text{ mm} \)

Resistance required/mm = \( \frac{117 \times 10^3}{500} = 234 \text{ N/mm} \)

Design resistance, \( F_{w,Rd} = f_{w,d} a \)

\( \therefore \) throat thickness required, \( a \geq 234/216 = 1.08 \text{ mm} \)

\( \therefore \) leg length required = \( a/0.7 = 1.08/0.7 = 1.54 \text{ mm} \)

Use a 6 mm fillet weld as a practical minimum.

### 1.9.6 Shear resistance of secondary beam web

**Influence of notch to the top flange**

The depth, \( d_n \), and length, \( c \), of the notch are standardised practical values for the section sizes which have been used\(^{12}\). Using these values, the problems of beam web stability in the region of the notch do not arise. For restrained beams in grade Fe 430 steel, the stability of the web need not be checked unless the length of the notch is greater than the beam depth.

Inspection shows that the moment due to the eccentricity of beam reaction is less than the moment resistance of the reduced beam section in the region of the notch.

\( \therefore V_{\text{sd}} \times (f_p + c) < M_{c,Rd \text{ (Reduced)}} \)

**Note** The secondary beam is fully restrained by the floor system.

If the beam were unrestrained, then the influence of the notch on the effective length used in the lateral torsional buckling calculation might need to be taken into account\(^{12}\).
1.9.7 Check bearing resistance of supporting beam
(Figure 19)

Figure 19 Web of the supporting primary beam (dimensions in mm)

Inspection shows that the bearing of the primary beam web is more critical than the bearing resistance of the secondary beam end-plates. (The beam web supports a pair of secondary beams.)

A detailed appraisal of the calculation of limiting bearing stresses is presented under Section 1.10 (Beam-to-column connection design). The limiting values stated here have been extracted from that calculation:

\[ F_{b,Rd} = 443.9 \text{ N/mm}^2 \text{ (limited bearing deformation)} \]

or

\[ F_{b,Rd} = 796 \text{ N/mm}^2 \text{ (substantial bearing deformation)} \]

The actual bearing stress:

\[ f_{b,\text{sd}} = \frac{2 V_{\text{sd}}}{4 d t_w} = \frac{2 \times 117 \times 10^3}{4 \times 20 \times 10.9} = 268.3 \text{ N/mm}^2 \]

\[ \text{i.e. } < 443.9 \text{ N/mm}^2 \]

\[ \therefore \text{ satisfactory.} \]

1.9.8 Tying force requirements

From Section 1.8, the secondary beams are required to resist a tying force of 117 kN to satisfy structural integrity requirements.

The ability of the connection to resist this force should be determined in accordance with Volume 1 of *Joints in simple construction*\(^{12}\), using an additional \(\gamma_r\) factor of 1.05 as explained in Section 1.10.9.

1.9.9 Design summary

Use four M20, grade 8.8 bolts and a 200 \(\times\) 250 \(\times\) 8 mm grade Fe 430 end-plate welded to the beam with a full-length 6 mm fillet weld (each side).
1.9.10 Design procedure using the concise document (C-EC3)²

This example can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following check in which a simpler procedure is used.

**Design end-plate weld**

In C-EC3 the calculation of $f_{ew}$ has been replaced by a table of design strengths.

Steel grade  =  Fe 430

$: f_{ew} = 216 \text{ N/mm}^2$

Total length of weld  $= 250 \times 2 = 500 \text{ mm}$

Resistance required/mm  $= 117 \times 10^{3}/500 = 234 \text{ N/mm}^2$

Throat thickness required  $= 236/216 = 1.08 \text{ mm}$

$: $leg length required  $= 1.09/0.7 = 1.54 \text{ mm}$

Use a 6 mm weld as a practical minimum,

$: \text{satisfactory.}$

References

C-EC3 Table 6.10

NAD 6.1.4 (g)
1.10 Beam-to-column connection (B2–C2)

1.10.1 Initial design information
Design the connection between the primary beam in Section 1.4 (533 × 210 × 101 UB, member reference B2) and an external column (254 × 254 × 73 UC, member reference C2) at the second-floor level.

From Section 1.4, the design ultimate beam reaction,
\[ V_{sd} = 242.5 \text{ kN} \]

From Section 1.8, the design beam tying force,
\[ T_F = 352 \text{ kN} \]

The connection will be nominally pinned and will be detailed so that rotation of the beam can occur without the connection attracting significant moments.

The connection will be designated as a 'Category A: Bearing type' shear connection, using two angle cleats to the beam web.

1.10.2 Basic details
To achieve the required flexibility, the connection (in Figure 20) will be detailed in accordance with Volume 1 of Joints in simple construction\(^2\).

![Diagram of beam-to-column connection]

**Figure 20** Connection details (dimensions in mm)
1.10.3 Check adequacy of bolts to the beam web

The design shear force is the vector sum of vertical and horizontal components$^2$. The horizontal component is that due to a moment arising from the support reaction acting through an eccentricity, $a$ (see Figure 20).

\[ F_{v,SD} = \left( F_v^2 + F_m^2 \right)^{0.5} \]

where $F_v$ is the vertical shear component per beam web bolt

\[ F_v = \frac{V_{sd}}{3} = 242.5/3 = 80.8 \text{ kN} \]

$F_m$ is the horizontal shear component per beam web bolt due to the reaction eccentricity

\[ F_m = \frac{V_{sd}}{a/Z_b} \text{ (for the outermost bolt)} \]

$Z_b$ is the elastic modulus of the bolt group

\[ Z_b = \frac{n (n + 1)}{6} \]

$p$ is the bolt pitch

$n$ is the number of bolts

\[ F_m = \frac{6 V_{sd} a}{n (n + 1) P} = \frac{6 \times 242.5 \times 60}{3 \times (3 + 1) \times 125} = 58.2 \text{ kN} \]

\[ F_{v,SD} = \left( 80.6^2 + 58.2^2 \right)^{0.5} = 99.4 \text{ kN} \]

Shear resistance of bolt per shear plane:

\[ F_{v,Rd} = \frac{0.6 f_{ub} A_s}{\gamma_{Mb}} \]

where $\gamma_{Mb} = 1.35$

$f_{ub}$ is the ultimate tensile strength of the bolt $= 800 \text{ N/mm}^2$

$A_s$ is the tensile stress area of the bolt $= 245 \text{ mm}^2$

\[ F_{v,Rd} = \frac{0.6 \times 800 \times 245}{1.35 \times 10^5} = 87.1 \text{ kN per shear plane} \]

For double shear, bolt resistance

\[ 2 \times 87.1 = 174.2 \text{ kN} \]

ie $> 99.4 \text{ kN}$, so the shear resistance of the bolts is adequate,

\[ \therefore \text{ satisfactory}. \]
1.10.4 Design for shear rupture resistance
(Figure 21)

Figure 21  Block shear failure of web

This check is applied to ensure that there is sufficient material remaining in the web allowing for the presence of bolt holes. It is seldom critical in connections which conform with standard UK detailing rules.

Shear resistance

\[ V_{\text{eff,Rd}} = (f_y/\sqrt{3}) A_{v,\text{eff}} \gamma_{M0} \]

where \( \gamma_{M0} = 1.05 \)

\[ A_{v,\text{eff}} = t \cdot L_{v,\text{eff}} \]

\[ L_{v,\text{eff}} = L_v + L_1 + L_2 \text{ (but } \leq L_3) \]

\[ L_v = 250 \text{ mm} \]

\[ L_1 = a_1 \text{ (but } \leq 5 \text{ d)} \]

\[ = 70 \text{ mm} \text{ (ie } < 5 \text{ d } = 100) \]

\[ L_2 = (a_2 - k \cdot d_{\alpha y}) (f_y/f_y) \]

For a single row of bolts, \( k = 0.5 \)

\[ L_2 = (50 - 0.5 \times 22) (430/275) = 61 \text{ mm} \]

\[ L_3 = L_v + a_1 + a_3 \]

\[ = 250 + 70 + 216.7 \]

\[ = 536.7 \text{ mm, but this should not be greater than} \]

\[ L_3 = (L_v + a_1 + a_3 - n \cdot d_{\alpha y}) (f_y/f_y) \]

\[ = (536.7 - 3 \times 22) (430/275) \]

\[ = 736 \text{ mm} \]

\[ L_{v,\text{eff}} = 250 + 70 + 61 \]

\[ = 381 \text{ mm} \]

\[ V_{\text{eff,Rd}} = (275/\sqrt{3}) 10.9 \times 381/1.05/10^3 \]

\[ = 628.0 \text{ kN} \]

ie > \( V_{sd} (= 242.5 \text{ kN}) \)

\[ \therefore \text{ satisfactory.} \]
1.10.5 Check bearing resistance of bolts

Bearing resistance:

\[ F_{b,rd} = \frac{2.5 \alpha f_y d t}{\gamma_{M_b}} \]

where \( \gamma_{M_b} = 1.35 \)
\( t = 10.9 \text{ mm} \)
\( d = 20 \text{ mm} \)
\( \alpha \) is the lesser of \( \left( \frac{e_i}{3 d_0} \right) \), \( \left( \frac{p_i}{3 d_0} - \frac{1}{4} \right) \), \( \frac{f_{ub}}{f_y} \) or 1.0
\( p_i \) is the bolt pitch = 125 mm
\( e_i \) is the distance of a bolt from a free edge in the direction of the applied load
\[ = \frac{99.4}{58.2} \times 50 \]
\[ = 85.4 \text{ mm} \]

**Note** The value of \( e_i \) has been chosen in accordance with the direction of the resultant shear force acting on the bolt.

\[ \therefore \frac{e_i}{3 d_0} = 1.29; \quad \frac{p_i}{3 d_0} - \frac{1}{4} = 1.64; \quad \frac{f_{ub}}{f_y} = 1.86 \]

\[ \therefore \text{minimum value is } \alpha = 1.0 \]

\[ \therefore \text{ultimate bearing strength} \]
\[ f_{b,rd} = \frac{2.5 \times 1.0 \times 430}{1.35} = 796 \text{ N/mm}^2 \]

This satisfies the Eurocode 3 requirement for bearing, but to avoid excessive hole deformation the NAD gives the following limit for the ultimate bearing stresses:

Limiting stress \[ \leq \frac{0.85 (f_y + f_s)}{\gamma_{M_b}} \]
\[ \leq \frac{0.85 (430 + 275)}{1.35} = 443.9 \text{ N/mm}^2 \]

**Note** This treatment of bearing stress in Eurocode 3 differs from that encountered in British Standard BS 5950: Part 1: 1990 \(^{13} \). The values stated in Table 33 of the British Standard ensure that little bearing deformation will occur, and are therefore similar to the NAD requirements. Eurocode 3, however, permits a larger bearing stress to be used, and takes advantage of the larger deformations which may occur in connection design methods where redistribution of forces between bolts is assumed.

Adopting the NAD limitation on bearing deformation, the bearing resistance per bolt is given by:

\[ F_{b,rd} = \text{limiting stress} \times d t \]
\[ = 443.9 \times 20 \times 10.9/10^3 \]
\[ = 96.8 \text{ kN} \]
ie \leq F_{v,5d} (99.4 \text{ kN})

For this particular connection, excessive vertical movement would be unacceptable. It is therefore necessary that the vertical component of bolt force be restricted to the NAD limit for satisfactory performance.

\[ F_v = 80.6 \text{ kN} \]

\[ ie \leq F_{v,rd} (96.8 \text{ kN}) \]

\[ \therefore \text{ satisfactory.} \]

1.10.6 Check bearing and shear resistance of web cleats

Inspection shows that this is a less critical design condition than that described for the bearing and shear of the beam web.

\[ \therefore \text{ satisfactory.} \]

1.10.7 Check resistance of bolts connecting cleat to column

The bolts would be checked under direct vertical shear only. There is no moment to be considered in this instance, and these are therefore no more critical than the bolts designed earlier, which connected the cleat to the beam.

\[ \therefore \text{ satisfactory.} \]

1.10.8 Check bearing resistance of column flange

254 \times 254 \times 73 \text{ UC section},

column flange thickness = 14.2 \text{ mm}

The flange thickness of the column is greater than the thickness of the beam web, so this is less onerous than the beam web bearing check carried out earlier.

\[ \therefore \text{ satisfactory.} \]

1.10.9 Check structural integrity of the connection

To satisfy the structural integrity requirements in the UK National Application Document, the connection is to resist a tie force, \( T_f \), in the absence of any other loading actions.

\[ T_f = 352 \text{ kN} \text{ (see main beam tying force, in Section 1.8)} \]

Check the adequacy of the connection using the procedure given in Volume 1 of Joints in simple construction\textsuperscript{15}. It should be noted that this is an empirically based method, and applies only to simple connections detailed in accordance with the reference. For this method to be consistent with Eurocode 3, a \( \gamma_m \) factor of 1.05 has been introduced, to allow for the difference between the \( \gamma_f \) factor of British Standard BS 5950\textsuperscript{13} and the \( \gamma_f \) factor of Eurocode 3.
Tension resistance of web cleats
For Fe 430 steel:
Tensile resistance = \(0.6 L_c t f_y/\gamma_M\)

where \(L_c = 2c + (n - 1)p - n d_o\)

- \(e\) is the end distance \(e_1\) = 35 mm
- \(p\) is the bolt pitch \(p_1\) = 125 mm
- \(n\) is the number of bolt rows = 3

\[L_c = 2 \times 35 + (3 - 1) \times 125 - 3 \times 22 = 254 \text{ mm}\]

\[\text{tensile resistance} = \frac{0.6 \times 254 \times 10 \times 275}{1.05 \times 10^3}\]

\[= 399 \text{ kN}\]

\(ie > 352 \text{ kN}\)

\(\therefore \text{satisfactory.}\)

Tension and bearing resistance of the beam web
(Figure 22)
Tensile resistance = \(L_c t f_y\)

where \(L_c = 2c + (n - 1)p - n d_o\)

- \(e\) is the edge distance \(e_2\) but \(c = 50 \text{ mm}\)
- \(p\) is the pitch \(p_1\) but \(2e = 100 \text{ mm}\)

\[L_c = 2 \times 50 + (3 - 1) \times 100 - 3 \times 22 = 234 \text{ mm}\]

\[\text{tensile resistance} = 234 \times 10.9 \times 275/1.05\]

\[= 668.0 \text{ kN}\]

\(ie > 352 \text{ kN}\)

\(\therefore \text{satisfactory.}\)

Check cleat to column bolts in the presence of large pry ing forces
Note: The method given in Eurocode 3 Annex J for determining pry ing forces is not intended to be used for angle cleat connections. Until such guidance is incorporated into Eurocode 3, the procedure given in Volume 1 of Joints in simple construction\(^{12}\), using British Standard BS 5950 principles, should be adopted.
Tension resistance of the bolt group \( A_t \) = 2 \( n \) \( A_t \) \( P_t \)/\( f_{yM} \)

where \( n \) is the number of bolt rows, = 3 (20 mm grade 8.8 bolts)

\( A_t \) is the tension area of bolt, = 245 mm\(^2\) (20 mm grade 8.8 bolts)

\( P_t \) is the reduced tension strength of bolts in the presence of extreme prying\(^1\), = 300 N/mm\(^2\) (20 mm grade 8.8 bolts)

Tension resistance \( = \frac{2 \times 3 \times 245 \times 300}{10^3 \times 1.05} \)

\( = 420 \text{ kN} \)

\( \therefore \) satisfactory.

1.10.10 Design procedure using the concise document (C-EC3)

This example can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following specific check in which a simpler procedure is used.

Bearing resistance of the supported beam

Here, C-EC3 considers a bearing strength coefficient, \( \beta \), which can be obtained directly from a table.

\( f_{b, Rd} = \beta \frac{d}{t} \gamma_{M_b} \)

where \( d = 20 \text{ mm} \)

\( t = 10.9 \text{ mm} \)

\( \gamma_{M_b} = 1.35 \)

\( \beta \) is the bearing strength coefficient

Where large hole deformations are not acceptable:

\( \beta \leq 0.6 \text{ kN/mm}^2 \) (for grade Fe 430)

\( e_1 = 85.5 \text{ mm} \)

\( \therefore \) \( e_1/d = 4.28 \)

\( \therefore \) where large hole deformations are acceptable,

\( \beta = 1.07 \text{ kN/mm}^2 \) (for grade Fe 430)

Limited hole deformation:

\( f_{b, Rd} = 0.6 \times 10^3/1.35 = 444 \text{ N/mm}^2 \)

Large hole deformation:

\( f_{b, Rd} = 1.07 \times 10^3/1.35 = 793 \text{ N/mm}^2 \)

The bearing resistance with limited hole deformation,\n
\( F_{b, Rd} = f_{b, Rd} \frac{d \times t}{10^3} = \frac{444 \times 20 \times 10.9}{10^3} = 96.8 \text{ kN} \)

\( \therefore \) satisfactory.
1.11 Column splice (C2–C3)

1.11.1 Initial design information
Column C3 provides support to the long span transfer beam B4 (see 'Plate girder', Section 1.6). Because of the high support reactions from this beam, the supporting column is assumed to be a 356 × 368 × 153 UC. This particular section will be provided from ground floor to 2nd floor, to distribute adequately the large moment due to the eccentricity of beam reaction. A column splice will be provided 0.5 m above the 2nd floor, at which point the section size will be reduced to a 254 × 254 × 73 UC (C2).

1.11.2 Load details
The characteristic axial forces in column C2–C3 are shown in Table 9

<table>
<thead>
<tr>
<th>Table 9 Loading for column C2–C3 (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>C2</td>
</tr>
<tr>
<td>Roof</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4th floor</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3rd floor</td>
</tr>
<tr>
<td>C3</td>
</tr>
<tr>
<td>2nd floor</td>
</tr>
<tr>
<td>1st floor</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

For column C2, levels 2 to 3, from Table 9:

\[ G_k = 409 \text{ kN} + s/w \text{ column} = 425 \text{ kN} \text{(say)} \]

\[ Q_k = 248 \text{ kN} \]

**Note** The imposed floor loading incorporates a reduction of 20% because of the number of floors which are supported.

Characteristic shear forces:

\[ V_{k} = 10 \text{ kN} \text{(say)} \]

\[ V_{qk} = 10 \text{ kN} \text{(say)} \]

**Note** Inspection shows that these nominal forces are sufficient to cater for horizontal elastic shears arising from nominal moments in the columns, and direct shear due to wind loading.

References

BS 6399: Part 1: 1984

Table 2
1.11.3 Basic details

(Figure 23)

As the column is being reduced by two serial sizes, it is inevitable that a substantial thickness of packing will be required. Under these conditions, it advisable to adopt a contact-in-bearing detail, with a division plate separating the upper and lower column sections (Figure 24). This avoids problems which can arise as a result of the substantial reduction in the shear resistance of bolts which pass through thick packing pieces. This reduction may prove critical when checking the tension resistance of the splice for structural integrity.

Figure 23 General arrangement of the splice detail (dimensions in mm)
Assuming a 45° load dispersion through the division plate, the minimum thickness of plate required is given by the greater of:

\[ \frac{d_1 - d_2}{2} \quad \text{(dispersion between outside faces)} \]

\[ \frac{d_1 - d_2}{2} + \frac{t_{12} - t_{11}}{2} \quad \text{(dispersion between inside faces)} \]

**Note**  The second equation governs where \( t_{12} > t_{11} \). That is not the condition in this example.

\[ t_p = \frac{d_1 - d_2}{2} = \frac{362 - 254}{2} = 54 \text{ mm} \]

\[ \therefore \text{ provide a 55 mm thick division plate.} \]

**Note**  This is the maximum thickness of plate for which no special measures are necessary to ensure adequate flatness for bearing in contact\(^1\).  

### 1.11.4 Determine the design loads

Because the edge beams have the same reaction both sides of the column:

Minor axis moment = 0 kNm

Major axis moment is a result of the eccentricity of reaction of beam B2. Centre of reaction to be taken as 100 mm from the face of the lower column.

\[ e = 100 + \frac{d_1}{2} = 100 + \frac{362}{2} = 281 \text{ mm} \]
Design reaction of beam B2 from Section 1.4:

$$V_{sd, beam} = 242.5 \text{kN}$$

Design applied moment:

$$M_{sd} = e \times V_{sd, beam} = 0.281 \times 242.5 = 68.1 \text{kNm}$$

This moment is shared between the upper and lower storeys, in proportion to the values of $$\left(\frac{L}{I}\right)_u$$ and $$\left(\frac{L}{I}\right)_u$$

\[
\left(\frac{L}{I}\right)_1 = \frac{11400 \times 10^4}{4000} = 2.85 \times 10^4
\]

\[
\left(\frac{L}{I}\right)_u = \frac{48500 \times 10^4}{4000} = 12.1 \times 10^4
\]

\[\therefore \text{moment distributed to the upper column} = \frac{2.85 \times 68.1}{2.85 + 12.1} = 13 \text{kNm}\]

Check whether this moment will induce any tensile forces in the splice.

Minimum compressive axial force in the column (permanent action only):

$$N_{sd} = \gamma_{G,inf} G_k$$

where $$\gamma_{G,inf} = 1.00$$

\[\therefore N_{sd} = 1.00 \times 425 = 425 \text{kN}\]

Tension will occur if:

$$M_{sd}/h \geq N_{sd}/2$$

where h can be assumed equal to the depth of the upper section

$$M_{sd}/h = 13.0/0.254 = 51.2 \text{kN}$$

$$N_{sd}/2 = 425/2 = 213 \text{kN}$$

ie > 51.2 kN

The splice therefore remains wholly in compression for all combinations of normal gravity loading.

The whole of the compressive force is transferred directly through bearing at the end of the column sections, and consequently the flange cover plates and associated fasteners are not required to carry load. Such connection elements are nevertheless essential to ensure a continuity of major and minor axis column stiffness. The plates should therefore have $$I_p$$, not less than 11 400 cm$^4$. Inspection shows that this requirement has been satisfied.

\[\therefore \text{satisfactory.}\]

Design shear force:

$$V_{sd} = \gamma_o G_k + \gamma_o Q_k$$

\[= 1.35 \times 10 + 1.5 \times 10 = 28.5 \text{kN}\]
The splice must be capable of resisting a minimum force, acting in any direction perpendicular to the member axis, equivalent to 2.5% of the axial force in the column.

Maximum force \( = 0.025 \times (1.35 \times 425 + 1.5 \times 248) \)
\[ = 23.6 \text{ kN}, \text{ ie } < 28.5 \text{ kN} \]

\( \therefore \) design shear force \( = 28.5 \text{ kN} \)

Although much of this shear force will be transferred by friction between the two sections, it is good practice to provide cleat fixings to the column webs to assist in the frame erection. It is evident by inspection that the fixings provided will be sufficient to resist this force.

\( \therefore \) satisfactory.

1.11.5 Check the integrity of the connection

It is a requirement in the UK National Application Document that column splices in 'tall' multi-storey buildings must be capable of resisting a tensile force based on the loading on the floor below.

Total ultimate load transferred to column C2 at level 2, from Table 9:
\[ = 1.35 \sum G_k + 1.5 \sum Q_k \]
\[ = 1.35 \times 134 + 1.5 \times 141 = 392 \text{ kN} \]

\( \therefore \) design tensile force \( = 2/3 \times 392 = 261.6 \text{ kN} \)

Inspection shows that the critical condition is the shear resistance of the bolts when reduced to take account of the packing plates, as already discussed.

Assume the tensile force is resisted by the flange plates only.

\( \therefore \) shear per bolt:
\[ F_{v,Sd} = \frac{261.6}{12} = 21.8 \text{ kN} \]
\[ F_{v,Rd} = \frac{0.6 f_{ub} A_s}{\gamma_{Mbd}} \]  
Table 6.5.3

where \( \gamma_{Mbd} = 1.35 \)
\( f_{ub} = 800 \text{ N/mm}^2 \)
\( A_s = 245 \text{ mm}^2 \)

\( \therefore \) \( F_{v,Rd} = \frac{0.6 \times 800 \times 245}{1.35 \times 10^3} = 87.1 \text{ kN} \)

Packing plate thickness, \( t_p > d/3 \)

Therefore reduce shear resistance for fasteners passing through packings.

Reduction factor, \( \beta_p = \frac{9 \times d}{8 d + 3 t_p} \), but \( \beta_p \leq 1 \)  
6.5.12 (1)

where \( d \) is the bolt diameter = 20 mm  
\( t_p \) is the packing thickness = 55 mm
\[ \beta_p = \frac{9 \times 20}{8 \times 20 + 3 \times 55} = 0.55 \]

\[ \therefore \text{reduced bolt resistance:} \]
\[ \beta_p F_{V,Rd} = 0.55 \times 87.1 = 47.9 \text{ kN} \]

\[ \therefore F_{V,Ed} (21.8 \text{ kN}) \]

\[ \therefore \text{ satisfactory.} \]

1.11.6 Design procedure using the concise document (C-EC3)

This example can also be designed using the concise version of Eurocode 3. In this instance, the design checks for C-EC3 are exactly the same as those for the Eurocode itself.
1.12 Column base-plate (C3)

A base-plate will be designed for column C3 as indicated in Figure 1.

1.12.1 Initial design information

Column section: 356 x 368 x 153 UC

Material strengths:
- Steel grade Fe 430: \( f_y = 275 \text{ N/mm}^2 \)
- Concrete (EC 2) grade C35/45: \( f_{ck} = 35 \text{ N/mm}^2 \)
- Grout \( f_{g,cube} = 12 \text{ N/mm}^2 \)

Note: Classification C35/45 refers to cylinder/cube strength.

Loadings

The characteristic axial loads on column C3 are shown in Table 9 in Section 1.11.

Characteristic axial:
Variable action:
Axial force in column C3 (ground to 1) \( N_{k,q} = 0.6 \times 1430.9 = 859 \text{ kN} \)

(The variable action incorporates a reduction of 40% because of the number of floors which are supported.)

Permanent action:
Axial load in column C3 (ground to 1) \( N_{k,g} = 1715.7 \text{ kN} \)

Characteristic shear:
The base shears resulting from the nominal moment induced in the column by the plate girder at level 1, are:

Due to variable action \( V_{k,q} = 679.3 \times (0.1 + 0.181) \times 0.56/4 = 27 \text{ kN} \)

Due to permanent action \( V_{k,g} = 831.2 \times (0.1 + 0.181) \times 0.56/4 = 33 \text{ kN} \)

where 0.56 allows for the distribution of the moment between 1st and 2nd floors

Foundation details

The column is supported concentrically on a 2100 x 2100 x 750 mm deep reinforced concrete pile cap.

Notes

- It will be specified that the column end shall be sawn cut square for tight bearing contact with the base-plate. Welds will not, therefore, be required to transmit axial compressive forces between the column and the base-plate, but welds will be provided to hold it in place and to resist the shear force.
- The column base-plate is not situated in a potentially corrosive environment. It is not necessary therefore to provide an all-round seal weld.

1.12.2 Design loading

Design compressive force:

\[
N_{sd} = 1.35 N_{k,g} + 1.5 N_{k,q} \\
= 1.35 \times 1715.7 + 1.5 \times 859 = 3605 \text{ kN}
\]

Design shear load:

\[
V_{sd} = 1.35 V_{k,g} + 1.5 V_{k,q} \\
= 1.35 \times 33 + 1.5 \times 27 = 85.0 \text{ kN}
\]
1.12.3 Axial resistance

Following the procedure in Annex L of Eurocode 3:

Column flange thickness, \( t_f = 20.7 \text{ mm} \)

The thickness of the base-plate should not be less than the thickness of the column flange.

Use a base-plate, thickness \( t > t_f \)

i.e \( 25 \text{ mm} \) (say).

Determine the maximum potential effective bearing width, \( c \), of the plate.

\[
c = t \left( \frac{f_y}{3 f_y \gamma_{60}} \right)^{0.5}
\]

where the bearing strength:

\[
f_y = \beta_j k_j f_{cd}
\]

where \( f_{cd} = f_{ck}/\gamma_c = 35/1.5 = 23 \text{ N/mm}^2 \) (for C35/C45 concrete)

\[
\beta_j = 0.67 \quad \begin{cases} f_{ck:cube} > 0.2 f_{ck:cube} \\ \text{grout thickness} < 0.2 \times 550 \end{cases}
\]

\( k_j \) is the concentration factor

\[
= \left( \frac{a_j b_j}{ab} \right)^{0.5}
\]
Figure 25 gives a view of the base-plate on foundation, where \( a = b = 550 \text{ mm} \) (say)

![Diagram of base-plate on foundation]

**Figure 25** Plan view of base-plate on foundation (dimensions in mm)

**Note** A 550 mm base-plate has been selected to provide adequate room for locating the holding-down bolts.

\[
\begin{align*}
    a_1 &= \text{lesser of:} \\
         &= a + 2a_2 = 550 + 2 	imes 775 = 2100 \text{ mm} \\
         &= 5a = 5 \times 550 = 2750 \text{ mm} \\
         &= a + h = 550 + 750 = 1300 \text{ mm}
\end{align*}
\]

\[\therefore a_1 = 1300 \text{ mm}\]

As the foundation and concentrically placed base-plate are both square, it is evident that:

\[b_1 = a_1 = 1300 \text{ mm}\]

\[\therefore k_j = \left( \frac{1300 \times 1300}{550 \times 550} \right)^{0.5} = 2.36\]

**Note** It is suggested in Eurocode 3 that to avoid these calculations a value of \( k_j = 1.0 \) can be used. It is evident, however, that this is a highly conservative simplification in many cases.
\[ f_y = 0.67 \times 2.36 \times 23 = 36.4 \text{ N/mm}^2 \]

\[ f_y = 275 \text{ N/mm}^2 \text{ (t < 40 mm)} \]

\[ \gamma_{M0} = 1.05 \]

\[ c = 25 \left( \frac{275}{3 \times 36.4 \times 1.05} \right)^{0.5} = 38.7 \text{ mm} \]

Figure 26 shows that the effective area of the base-plate is based on a cantilever projection of 38.7 mm from the column. The bearing stresses are therefore distributed in an I-shaped profile. This differs from the approach in British Standard BS 5950: Part 1, in which the bearing stresses are assumed to be evenly distributed over a rectangular profile\(^{13}\).

![Diagram of effective area](image)

**Figure 26** Area of plate effective in resisting axial compression (dimensions in mm)

Effective area, \( A_{eff} = (362.0 + 2 \times 38.5)(370.2 + 2 \times 38.5) - 2 \times 243.6 \times 178.8 \)

\[ = 109.2 \times 10^6 \text{ mm}^2 \]

Design bearing pressure \( = \frac{N_{bd}}{A_{eff}} \)

\[ = \frac{3605 \times 10^3}{109.7 \times 10^3} = 32.9 \text{ N/mm}^2 \]

Bearing strength \( = f_y = 36.8 \text{ N/mm}^2 \)

\[ \text{ie} > 32.9 \text{ N/mm}^2 \]

\[ \therefore \text{ satisfactory.} \]
1.12.4 Shear resistance
Where the applied shear force is less than 20% of the applied vertical load, no special provisions are necessary for the transfer of the shear load from the base-plate to the foundation.\(^1\)

\[
N_{sd} = 3605 \text{ kN} \quad \therefore \quad N_{sd}/5 = 0.2 \times 3605 = 721.0 \text{ kN}
\]

\[
V_{sd} = 85 \text{ kN}
\]

\[i.e. \quad < 721.0 \text{ kN}\]

\[\therefore \quad \text{satisfactory.}\]

1.12.5 Determine plate dimensions
Minimum width of plate required
\[
= b + 2c = 370.2 + 2 \times 38.5 = 447.2 \text{ mm}
\]

Minimum depth of plate required
\[
= d + 2c = 362.0 + 2 \times 38.5 = 439.0 \text{ mm}
\]

but a dimension of 550 mm is needed to accommodate the holding-down (HD) bolts,

\[\therefore \quad \text{use a } 550 \times 550 \times 25 \text{ mm thick grade Fe 430 base-plate. (See Figure 27.)}\]

1.12.6 Welding requirements
The plate and column are in tight bearing. There is, therefore, no need to check the ability of the weld to transmit axial loads. However, sufficient weld should be used along the web of the section to allow the safe transfer of the applied shear force. In the case of the flanges, these will be provided with a full length weld on the outer face, in accordance with good detailing practice.

\[
V_{sd} = 85 \text{ kN}
\]

Weld shear strength:

\[
f_{w,d} = \frac{f_u}{\beta_w \gamma_{uw} \times \sqrt{3}}
\]

\[
f_u = 430 \text{ N/mm}^2 \quad \text{grade Fe 430}
\]

\[
\beta_w = 0.85 \quad \text{grade Fe 430}
\]

\[
\gamma_{uw} = 1.35 \quad \text{grade Fe 430}
\]

\[\therefore \quad f_{w,d} = \frac{430}{0.85 \times 1.35 \times \sqrt{3}} = 216.3 \text{ N/mm}^2\]

Using an 8 mm fillet weld:

\[
\text{Throat thickness, } a = 0.7 \times 8 = 5.6 \text{ mm}
\]

Resistance of weld/mm:

\[
F_{w,8d} = f_{w,d} a = 216.3 \times 5.6 = 1.21 \times 10^3 \text{ N/mm}
\]

\[\therefore \quad \text{length of weld required} = \frac{107.1 \times 10^3}{1.21 \times 10^3} = 88.5 \text{ mm}\]
Note: There is no reduction in Eurocode 3 for weld runs which are 'not returned'. Instead, only the length of full-size fillet is measured. It should be noted that the minimum such length is 40 mm or six times the throat thickness.

Use a 100 mm run of 8 mm weld on each side of the column web.

Total weld run = 200 mm,

ie > 88.5 mm

∴ satisfactory.

1.12.7 Design summary

(Figure 27)

Figure 27 Summary of base-plate details (dimensions in mm)
1.12.8 Design procedure using the concise document (C-EC3)²

This example can also be designed using the concise version of Eurocode 3. The procedure is similar to that given in the Eurocode itself, except for the following checks in which a simpler procedure is used.

Welding requirements
In C-EC3 the calculation of \( f_{vw} \) has been replaced by a table of design strengths.

\[
V_{sd} = 107.1 \text{kN}
\]

Steel grade Fe 430

\[
\therefore f_{vw} = 216 \text{N/mm}^2
\]

Using an 8 mm fillet weld:

Throat thickness, \( a = 0.7 \times 8 = 5.6 \text{mm} \)

\[
F_{w, Rd} = f_{vw} a
\]

\[
= 216 \times 5.6 = 1.21 \times 10^3 \text{N/mm}^2
\]

\[
\therefore \text{length of weld required } = \frac{107 \times 10^3}{1.21 \times 10^3} = 88.5 \text{mm}
\]

Use a 100 mm run of 8 mm weld on each side of the column web.

Total weld run = 200 mm, \( ie > 88.5 \text{mm} \)

\[
\therefore \text{satisfactory.}
\]
Example 2
Design of continuous multi-storey frames

Introduction
This example covers the design of a 2-storey rigid frame for the sway failure mode, by considering the vertical load to act in conjunction with the horizontal loading, including the effects of sway imperfections. The second-order effects associated with sway deformation are also considered.

Although not included in this example, sway frames should first be checked for the non-sway failure mode, using appropriate buckling lengths for the columns under the most unfavourable loading conditions, including pattern loading.

For the purposes of this example, all the beams are $610 \times 229 \times 113$ UBs, and all columns are $305 \times 305 \times 158$ UC sections.

Internal forces and moments are determined from an elastic global analysis of the structure. Frame imperfections are considered in accordance with the requirements of Eurocode 3: Part 1.1. All joints in the frame are rigid, apart from the feet, which are assumed to be pinned.

Plastic analysis
This frame could be analysed for the sway mode using plastic methods, but has not been, for the following reasons:
• The plastic moment resistance of the columns depends on, and is sensitive to, the axial load in the columns, which means that iteration is necessary.
• The amplification effects resulting from the sway behaviour of the frame depend upon the sizes of the members
• One of the advantages of plastic analysis is normally that it is possible to select the member sizes after completing the analysis, but for a sway frame this advantage is lost.

If the frame were to be braced against side-sway, then simple plastic analysis would be the quickest method of designing the frame as a continuous structure.

Analysis of the frame
The frame may be analysed using any suitable method, including hand methods, to determine accurately the resulting actions and deflections. Normally, for frames of this complexity, an analysis would be undertaken using a computer program. For the purposes of this example, a linear elastic analysis is used. Most programs will also check the stability of the structure during analysis, but in this example the stability has been checked by hand to demonstrate the rules in the Eurocode.
2.1 Frame geometry, loading and analysis

The example is based on a typical frame, taken from the centre of a long building, shown in the cross-section in Figure 28 and part-plan in Figure 29.

![Diagram of frame geometry, loading and analysis](image)

*Figure 28* Section through building showing rigid frame (dimensions in mm)

![Diagram of part-plan showing layout of floor beams](image)

*Figure 29* Part-plan showing layout of floor beams (dimensions in mm)

Apart from the provision of moment-resisting connections instead of cross-bracing, the form of construction is similar to that given in Example 1, with the secondary beams having the same loading and size as those given earlier. The remainder of the loading is similar to that in Example 1, except that the wind is taken as having a dynamic pressure of 0.45 kN/m² and the coefficient $C_v$ is taken as 1.1.
In this example, both lateral wind loads and gravity loads are acting on the frame. These must be combined. In principle, the wind load and the gravity load should each be treated in turn as the dominant load, and the worst case derived from an analysis of the structure. In practice, it will be found that for low-rise structures the gravity loading will dominate the design. This will, therefore, be taken as the dominant loading in the analysis which follows. It is assumed that the designer will also carry out the checks needed for serviceability.

2.1.1 Frame imperfections
Imperfections in the frame are considered by taking a closed series of horizontal forces on the frame, derived from the imperfections, which are not to be included in the net shear applied to the foundations. The imperfections are derived from the following formula:

\[ \phi = k_c k_s \phi_o \]

where \( k_c = (0.5 + 1/n_c)^{0.5} \) but \( k_c \leq 1.0 \)
\( n_c \) is the number of full-height columns per plane = 3
\( k_s = (0.2 + 1/n_s)^{0.5} \)
\( n_s \) is the number of storeys in the structure = 2
\( \phi_o = 1/200 \)
\[ k_e = (0.5 + 1/3)^{0.5} = 0.913 \]
\[ k_i = (0.2 + 1/2)^{0.5} = 0.837 \]
\[ \therefore \phi = 0.913 \times 0.837/200 = 1/262 \]

The horizontal load at each point in the structure should be derived from the design vertical load multiplied by 1/262.

2.1.2 Summary of loading
The characteristic and design values of the loads are summarised in Table 10.

| Table 10 | Vertical loads (kN) |
|----------|---|---|---|
| Load | \( \gamma_F \) | Design (kN) | 0.9 design* (kN) |
| **Roof level** |  |  |
| Edge beams: |  |  |
| Variable actions | 1.5 \( \times \) 2.5/2 \( \times \) 7.5 | = 14.1 | 1.50 | 21.1 | 19.0 |
| Permanent actions | 5 \( \times \) 2.5/2 \( \times \) 7.5 | = 46.9 | 1.35 | 63.3 |
| Internal beams: |  |  |
| Variable actions | 1.5 \( \times \) 2.5 \( \times \) 7.5 | = 28.1 | 1.50 | 42.2 | 38.0 |
| Permanent actions | 5 \( \times \) 2.5 \( \times \) 7.5 | = 93.8 | 1.35 | 126.6 |
| **Floor level** |  |  |
| Edge beams: |  |  |
| Variable actions | 5 \( \times \) 2.5/2 \( \times \) 7.5 | = 46.9 | 1.50 | 70.4 | 63.4 |
| Permanent actions | 3.7 \( \times \) 2.5/2 \( \times \) 7.5 | + 0.8 \( \times \) 4 \( \times \) 7.5 | = 58.7 | 1.35 | 79.2 |
| Internal beams: |  |  |
| Variable actions | 5 \( \times \) 2.5 \( \times \) 7.5 | = 93.8 | 1.50 | 140.6 | 126.5 |
| Permanent actions | 3.7 \( \times \) 2.5 \( \times \) 7.5 | = 69.4 | 1.35 | 93.7 |

* This column is for use with formula 2.12 covering the simplified load combinations.
The design values of the wind and equivalent horizontal loads are summarised in Table 11.

<table>
<thead>
<tr>
<th>Location</th>
<th>Vertical (kN)</th>
<th>Horizontal (kN)</th>
<th>0.9 horizontal* (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable actions</td>
<td>21.1 x 2 + 42.2 x 6 = 295.4</td>
<td>1.13</td>
<td>1.02</td>
</tr>
<tr>
<td>Permanent actions</td>
<td>63.3 x 2 + 126.6 x 6 = 886.2</td>
<td>3.38</td>
<td></td>
</tr>
<tr>
<td>Wind</td>
<td>6.75</td>
<td>6.08</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11.26</td>
<td>10.48</td>
<td></td>
</tr>
<tr>
<td>Floor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable actions</td>
<td>35.2 x 2 + 140.6 x 6 = 914.0</td>
<td>3.49</td>
<td>3.14</td>
</tr>
<tr>
<td>Permanent actions</td>
<td>79.2 x 2 + 93.7 x 6 = 720.6</td>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td>Wind</td>
<td>13.50</td>
<td>12.15</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>10.75</td>
<td>18.04</td>
<td></td>
</tr>
</tbody>
</table>

* This column is for use with formula 2.12 covering the simplified load combinations.

2.1.3 Load combinations

Loads should be combined to give the most unfavourable effects on the various parts of the structure, assuming reasonable load patterns. Typical layouts which give the most critical cases are shown in Figure 30 for the vertical loads.

![Diagram showing load combinations](image)

X: Point where critical moment will be found

Point where critical axial load in column will be found

These load patterns do not necessarily determine the critical case for moments in the columns

Figure 30 Loading patterns for full analysis

For each load case the imposed floor, imposed roof and the wind load can, in principle, be taken as the dominant action in turn. For each of these conditions the wind loads and loads from the frame imperfections should be considered as either acting from the left or acting from the right.

This would give a total of 5 x 3 x 2 = 30 load cases

This is clearly not a practical proposition for a frame of any complexity. Eurocode 3 therefore gives an alternative load case which may be used for practical design. It allows a single load combination to be considered for each group of loading, and a reduced combination factor for multiple loading cases. These simplified combinations are used in the example.
The following load cases will be considered:

1. Dead load plus wind and imperfections acting to the right
2. Dead and 0.9 × all imposed loads plus wind and imperfections acting to the right
3. Dead and full imposed floor loads and imperfections acting to the right
4. Dead and full imposed roof loads and imperfections acting to the right
5. Dead load plus wind and imperfections acting to the left
6. Dead and 0.9 × all imposed loads plus wind and imperfections acting to the left
7. Dead and full imposed floor loads and imperfections acting to the left
8. Dead and full imposed roof loads and imperfections acting to the left

When considering the effects of vertical load it will be found that a frame with the asymmetry present here will naturally sway to the right: this may be used to reduce the number of cases considered.

2.1.4 Sway stability check

The stability is investigated by checking whether the loading on the frame is less than 10% of the elastic critical load \( V_{sd}/V_{cr} < 0.1 \). In the event of this being greater than 0.1, the frame is defined as a sway frame and must be checked accordingly. If the frame is not a sway frame, the forces may be taken directly from the elastic analysis, and the effective lengths of columns may be determined from Figure E.2.1 of Eurocode 3: Part 1.1.

To determine the value of \( V_{sd}/V_{cr} \), the code gives the following relationship:

\[
V_{sd}/V_{cr} = (\delta/h) (V/H)
\]

where \( \delta \) is the horizontal displacement of the top of the storey, relative to the bottom of the storey, from both vertical and horizontal loads

- \( h \) is the storey height
- \( H \) is the horizontal reaction at the bottom of the storey including applied and notional actions
- \( V \) is the total vertical reaction at the bottom of the storey

**Note** \( V_{sd}/V_{cr} \) is the reciprocal of the critical load factor, \( \lambda_{cr} \), used in British Standard BS 5950. In BS 5950, \( \delta \) is defined as the displacement from horizontal loads only. The codes therefore give similar results for symmetrical frames with symmetrical loading, because in these situations there are no horizontal displacements from vertical loads.

In BS 5950, \( \delta \) is defined as the displacement from horizontal loads only. The codes therefore give similar results for symmetrical frames with symmetrical loading, because in these situations there are no horizontal displacements from vertical loads.

For asymmetric frames or asymmetric loading, the difference can lead to much heavier sections being required using Eurocode 3. For this example, the Eurocode 3 definition leads to 305 × 305 × 158 UC grade Fe 430 columns, while the British Standard BS 5950 definition would have led to 254 × 254 × 107 UC grade Fe 430 columns.

The Eurocode 3 definition will be reviewed during the ENV period, as it is believed to give conservative results for asymmetrical frames. This example, however, uses the current definition.
The sway check information is tabulated in Table 12.

### Table 12  Sway stability check

<table>
<thead>
<tr>
<th>Case</th>
<th>Storey</th>
<th>$\delta$ (mm)</th>
<th>$V$ (kN)</th>
<th>$H$ (kN)</th>
<th>$5\delta h^* V/H$</th>
<th>Frame type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Top</td>
<td>2.5</td>
<td>886.2</td>
<td>10.1</td>
<td>0.05</td>
<td>Non-sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>4.9</td>
<td>1606.8</td>
<td>26.4</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Top</td>
<td>3.5</td>
<td>1152.2</td>
<td>10.5</td>
<td>0.10</td>
<td>Sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>6.5</td>
<td>2758.6</td>
<td>28.5</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Top</td>
<td>2.6</td>
<td>886.2</td>
<td>3.4</td>
<td>0.17</td>
<td>Sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>3.8</td>
<td>2591.2</td>
<td>9.6</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Top</td>
<td>2.3</td>
<td>1181.6</td>
<td>4.5</td>
<td>0.15</td>
<td>Sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>2.0</td>
<td>1902.2</td>
<td>7.3</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Top</td>
<td>0.6</td>
<td>886.2</td>
<td>10.1</td>
<td>0.01</td>
<td>Non-sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>3.4</td>
<td>1606.8</td>
<td>26.4</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Top</td>
<td>1.6</td>
<td>1152.2</td>
<td>10.5</td>
<td>0.04</td>
<td>Non-sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>2.7</td>
<td>2758.6</td>
<td>28.5</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Top</td>
<td>2.0</td>
<td>886.2</td>
<td>3.4</td>
<td>0.13</td>
<td>Sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>0.9</td>
<td>2591.2</td>
<td>9.6</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Top</td>
<td>1.6</td>
<td>1181.6</td>
<td>4.5</td>
<td>0.11</td>
<td>Sway</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td>0.7</td>
<td>886.2</td>
<td>7.3</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

Height of top storey, $h = 4000$ mm
Height of bottom storey, $h = 4300$ mm

In checking the members for load cases 2, 3, 4, 7 and 8, the effects of sway must be considered. In all other load cases the frame is not considered sensitive to sway, and the forces on the members may be taken directly from an elastic analysis.

To check the frame for sway, the Eurocode gives three methods. These are:

- The use of rigid plastic analysis, which will require a computer capable of carrying out the stability checks (This method will not be given here.)
- First order elastic analysis, with amplified sway moments
- First order elastic analysis, with sway mode buckling lengths

In this example the second method is used.

This approach may be used for frames, provided that the value of $V_{sd}/V_{cr} < 0.25$, which is the case in this example. 5.2.6.2 (4)

For this method the sway moments are amplified and added to the remainder of the moments in the frame. The sway moments result from horizontal applied loads, equivalent horizontal forces and the effects of sway if the frame or the vertical loading are asymmetrical.
The amplification factors for each load case are given in Table 13.

### Table 13  Amplification factors (α)

<table>
<thead>
<tr>
<th>Load case</th>
<th>$V_{sd}/V_{cr}$</th>
<th>$\alpha = \frac{1}{1 - V_{sd}/V_{cr}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>1.18</td>
</tr>
<tr>
<td>3</td>
<td>0.24</td>
<td>1.32</td>
</tr>
<tr>
<td>4</td>
<td>0.12</td>
<td>1.14</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.13</td>
<td>1.14</td>
</tr>
<tr>
<td>8</td>
<td>0.11</td>
<td>1.12</td>
</tr>
</tbody>
</table>

The horizontal loads and equivalent horizontal forces may be obtained from Table 11. The appropriate value of the amplification factor $\alpha$ should be obtained from Table 13.

The sway moments due to asymmetry of the frame or the vertical loading can be accounted for by proceeding as follows.

1. Analyse the frame under the vertical loading only, with the floor levels restrained against lateral movement, and determine the lateral reactions at those restraints.

2. Analyse the frame (without the floor levels restrained against lateral movement) under the following loads:
   - the vertical loads;
   - the sum of the applied horizontal loads and the equivalent horizontal forces, all multiplied by $\alpha$; and
   - a series of lateral forces equal and opposite to the lateral reactions found in (1), multiplied by $(\alpha - 1)$. 

References
2.2 Beam design

The beam selected in this example is the 10 m span beam on the first-floor level, shown in Figure 31, with the final design moments.

From the analysis, load case 3 is the most critical load case for this beam, and Figure 31 shows its bending moment diagram.

To design the beam, the maximum moments and forces should be determined and the member checked to ensure that its resistance is adequate. This will involve checking the moments at the supports and mid-span of the section.

![Beam moments for design (kNm)](image)

Eurocode 3 allows the designer to modify the moments by redistributing up to 15% of the peak moment in a member, provided that the sections are at least class 2 and that equilibrium is maintained. For members of the uniform cross-section, the general effect is to reduce the moments at the supports and increase those in the span, but it is illogical to reduce the support moment to the extent that the increased span moments exceed the reduced support moments. In this case the right-hand support moment is reduced by 15%, but the left-hand support moment is not the peak moment so cannot be reduced. Table 14 shows the values for the adjusted moments.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Axial (kN)</th>
<th>$M_A$ (kNm)</th>
<th>$M_B$ (kNm)</th>
<th>$M_C$ (kNm)</th>
<th>$M_D$ (kNm)</th>
<th>$M_E$ (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>135</td>
<td>-412</td>
<td>393</td>
<td>583</td>
<td>160</td>
<td>-878</td>
</tr>
<tr>
<td>Adjusted</td>
<td>-412</td>
<td>426</td>
<td>649</td>
<td>259</td>
<td>-746</td>
<td></td>
</tr>
</tbody>
</table>

Table 14 Moments on beam
Following the adjustment all shear forces and axial forces must be recalculated to maintain equilibrium. In this case, the axial force in this beam is assumed to be unchanged.

The beam will be checked for the adjusted moments and a tensile force of 135 kN. In this frame the floor beams are in tension and the roof beams are in compression. It is essential that the designer check the type of axial force in the member. The maximum shear is taken as 415 kN, the maximum from any of the load cases, and is not normally critical. If the maximum shear were high enough to reduce the moment resistance of the section, a more detailed analysis using the actual moments and shears at coincident points should be undertaken.

### 2.2.1 Properties of the selected section

The selected section, which was used for the analysis of the frame, is a 610 × 229 × 113 UB grade Fe 430, with the following properties:

- \( h = 607.3 \text{ mm} \)
- \( b = 228.2 \text{ mm} \)
- \( t_w = 11.2 \text{ mm} \)
- \( t_f = 17.3 \text{ mm} \)
- \( d = 547.3 \text{ mm} \)
- \( d/t_w = 48.9 \)
- \( c/t_f = 6.60 \)
- \( W_{pl} = 3290 \text{ cm}^3 \)
- \( W_{pl} = 2880 \text{ cm}^3 \)
- \( I_x = 246 \text{ mm} \)
- \( i_x = 48.8 \text{ mm} \)
- \( A = 14400 \text{ mm}^2 \)
- \( i_{LT} = 55.5 \text{ mm} \)
- \( a_{LT} = 1630 \text{ mm} \)

**Note** All these parameters can be obtained from Eurocode 3 section tables.

**Material properties**

The flange thickness is less than 40 mm,

\[
.: f_y = 275 \text{ N/mm}^2
\]

### 2.2.2 Section classification

The section is in tension and bending. It will be checked as though there were only bending present. This is slightly conservative in this case. It should be noted that the section is in an elastically designed frame, with moment redistribution. This means that the section must be at least class 2.

\[
\epsilon = (235/f_y)^{0.5} = (235/275)^{0.5} = 0.924
\]

(a) Flange check \( c/t_f = 6.6 \)

\[
10 \epsilon = 10 \times 0.924 = 9.2 \geq 6.6
\]

(b) Web check \( d/t_w = 48.9 \)

\[
72 \epsilon = 72 \times 0.924 = 66.5 \geq 48.9
\]

\[
.: \text{ all elements are class 1.}
\]
2.2.3 Shear resistance

Maximum shear on the section at any point for all load cases is 415 kN.

The shear area $A_s$ may be calculated in two ways, the simplest method being slightly conservative. As the effects of shear on the web are normally not a design limit, the simpler method may be used, as in this example.

Shear area $A_s = 1.04 \times t_w = 1.04 \times 607.3 \times 11.2 = 7074 \text{ mm}^2$

Shear resistance $V_{p/Re} = \frac{A_s (f_e/\gamma_M)}{\gamma_M}$

$= \frac{7074 (275/\gamma_M)}{1.05 \times 10^3} = 1070 \text{ kN}$

$V_{sd}/V_{p/Re} = 415/1070 = 0.388$

As this is less than 0.5 it will have no effect on the moment resistance of the section.

$\therefore$ satisfactory.


2.2.4 Resistance of cross-section

In this case the floor provides no restraint to the top flange, and the critical section is that between D and E in Figure 31. This section has the highest moment on the beam and is the critical length in this example. However, in practice any other length that may be critical should also be checked. In this case the length B-C has a less favourable shape of bending moment diagram and is almost as critical. It has already been shown that the maximum shear on the section will not affect the moment resistance, so this will not be considered further.

The section has a tensile force, as well as the moment. This is relatively small and its effect will be checked first.

Eurocode 3 states that the axial load need not be considered as making a significant reduction in the resistance moment of a class 1 or class 2 cross-section, due to strain hardening, as long as two conditions are satisfied. These are (1) that the axial load be less than half the axial resistance of the web, and (2) that the axial force be less than a quarter of the axial resistance of the whole section. These two conditions will be checked.

(1) Axial resistance of the web $= f_y \times t_w / \gamma_M$

$= \frac{275 \times 607.3 \times 11.2}{1.05 \times 10^3} = 1781 \text{ kN} \quad 135 \text{ kN} < \frac{1781}{2} \text{ kN}$

(2) Axial resistance of the section $= f_y \times A / \gamma_M$

$= \frac{275 \times 14400}{1.05 \times 10^3} = 3771 \text{ kN} \quad 135 \text{ kN} < \frac{3771}{4} \text{ kN}$

As a result, the effects of the axial load need not be considered further.


2.2.5 Resistance of member

The moment resistance of the member, allowing for lateral torsional buckling, must now be determined. This requires the determination of $\chi_{LT}$ using $\lambda_{LT}$ and curve a in Table 5.5.2, where $\lambda_{LT}$ is calculated from $\lambda_{LT}$. Table 5.5.2
\[ \psi = -259/746 = -0.347 \]
\[ \therefore C_1 = 1.88 - 1.40 \times 0.347 + 0.52 \times 0.347 \]
\[ C_1 = 2.429 \]

\[ \dot{\lambda}_{LT} = \frac{L/h_{LT}}{(C_1)^{0.5} \left[ 1 + \left( \frac{L/\lambda_{LT}}{25.66} \right)^{0.25} \right]} \]
\[ = \frac{2500/55.5}{2.429 \left[ 1 + \left( \frac{2500/1630}{25.66} \right)^{0.25} \right]} = 28.3 \]

\[ \ddot{\lambda}_{LT} = \lambda_{LT} \lambda_1 \left( \beta_w \right)^{0.5} \]
where \[ \lambda_1 = 93.9 \epsilon = 93.9 \times 0.924 = 86.8 \]

\[ \therefore \ddot{\lambda}_{LT} = 28.3/86.8 = 0.326 \]

If \[ \ddot{\lambda}_{LT} < 0.4 \], then \[ \dot{\lambda}_{LT} \] may be taken as 1.0.

The member will therefore have a resistance equal to the full resistance moment of the cross-section.

\[ M_{pRd} = f_y W_{pRd} / \gamma_{M0} = \frac{275 \times 3290}{1.05 \times 10^3} = 862 \text{ kNm} \]

This is greater than the maximum applied moment \( M_{kD} = 746 \text{ kNm} \) on the length D-E. (As already noted, the length B-C should also be checked in practice. See Figure 31.)

### 2.2.6 Deflection

From the elastic analysis, the maximum deflection under ultimate load is 20.4 mm from load case 3.

\[ \therefore \text{Deflection} \quad \frac{20.4}{10000} = \frac{1}{490} \]

Comparing this ratio to the recommended values for vertical deflection in Table 4.1 of Eurocode 3: Part 1.1, shows by inspection that there is no need to consider serviceability loads and deflections.
2.3 Column design

2.3.1 Actions
The maximum axial forces and moments occur in the centre column and are from load case 6. Figure 32 shows the distribution of axial force and major axis moments in the centre column.

![Distribution of forces in the centre column](image)

Figure 32 Distribution of forces in the centre column

The maximum shear in the column is 132 kN.

2.3.2 Section properties
Check 305 × 305 × 158 UC grade Fe 430.

- \( h = 327.2 \text{ mm} \)
- \( t_w = 15.7 \text{ mm} \)
- \( r = 15.2 \text{ mm} \)
- \( d/t_w = 15.7 \)
- \( A = 201 \text{ cm}^2 \)
- \( W_{P_{ef}} = 2370 \text{ cm}^3 \)
- \( W_{P_{ef}} = 2680 \text{ cm}^3 \)
- \( i_y = 13.9 \text{ cm} \)
- \( i_x = 7.89 \text{ cm} \)
- \( a_{LT} = 869 \text{ mm} \)

Note All these properties, including \( i_{LT} \) and \( a_{LT} \), can be obtained from Eurocode 3 section tables³.

Material properties
- \( t_w \) and \( t_p \) ≤ 40 mm

\[ \therefore f_y = 275 \text{ N/mm}^2 \]
2.3.3 Section classification

\[
\varepsilon = \left( \frac{235}{f_y} \right)^{0.5}
\]

\[
= \left( \frac{235}{275} \right) = 0.924
\]

\[
33\varepsilon = 33 \times 0.924 = 30.5 \geq 15.7
\]

\[
10\varepsilon = 10 \times 0.924 = 9.24 \geq 6.21
\]

\[\therefore \text{all elements are at least class 1}\]

\[\therefore \text{cross-section is class 1.}\]

2.3.4 Check shear

\[
A_s = A - 2b t_t + (t_w + 2r) t_t
\]

\[
= 20 \times 100 - 2 \times 310.6 \times 25.0 + (15.7 + 2 \times 15.2) 25.0
\]

\[
= 5722 \text{ mm}^2
\]

\[
V_{p/Rd} = \frac{A_s f'/\sqrt{3}}{Y_{t0}}
\]

\[
= \frac{5722 \times 275 \sqrt{3}}{1.05 \times 10^3} = 865 \text{ kN}
\]

\[
V_{sd} = 132 \text{ kN}
\]

\[
\frac{V_{sd}}{V_{p/Rd}} = \frac{132}{865} = 0.153 \leq 0.5
\]

\[\therefore \text{no reduction need be made to the moment resistance of the section.}\]

2.3.5 Resistance of the cross-section

Local resistance, bending and axial force

For class 1 and class 2 cross-sections the criterion to be satisfied is:

\[
M_{sod} \leq M_{Ny,Rd}
\]

for standard rolled sections

\[
n = \frac{N_{sd}}{N_{p/Rd}} = \frac{1552 \times 10^3}{201 \times 10^3 \times 275/1.05} = 0.295
\]

\[
M_{Ny,Rd} = 1.11 \left(1 - n\right) M_{p/Rd}
\]

\[
= 1.11 \times \left(1 - 0.295\right) \times \frac{2680 \times 10^3 \times 275}{1.05}
\]

\[
= 549 \text{ kNm}
\]

This is greater than the applied moment (299 kNm),

\[\therefore \text{satisfactory.}\]
2.3.6 Flexural buckling

Centre column: ground floor to 1st floor

The section should satisfy the following criterion:

$$\frac{N_{St}}{\chi_{\min} A f_y / f_{y1}} + \frac{k_y M_{St}}{W_{p,ty} f_y / f_{y1}} \leq 1.0$$

To calculate $\chi_{\min}$

$\chi_{\min}$ is the lesser of $\chi_y$ and $\chi_z$, where $\chi_y$ and $\chi_z$ are the reduction factors from clause 5.5.1 for the y-y and z-z axes respectively.

**Determination of $\chi_y$**

The reduction factor $k_y$ depends on the slenderness about the y-y axis. The effective length in the plane of the frame is taken as the system length.

$$\ell_y = L = 4300 \text{ mm}$$

$$\therefore \lambda_y = \ell_y / i_y = 4300 / 139 = 30.9$$

$$\lambda_1 = 93 \varepsilon = 86.8$$

$$\bar{\lambda}_y = \lambda_y / \lambda_1 \times \beta_{A.05} = 30.9 / 86.8 \times 1.0^{0.5} = 0.356$$

From Table 5.5.3 use buckling curve b.

$$\therefore \chi_y = 0.945$$

**Determination of $\chi_z$**

It is assumed that the frame is braced out of plane and that the connections of the secondary beams have sufficient stiffness to limit the rotation of the columns about their weak axis. The buckling length may be taken as $0.85 \times$ system length.

$$\ell_z = 0.85 \times 4300 = 3655 \text{ mm}$$

$$\lambda_z = \ell_z / i_z = 3655 / 78.9 = 46.3$$

$$\bar{\lambda}_z = \lambda_z / \lambda_1 \times \beta_{A.05} = 46.3 / 86.8 = 0.534$$

Using curve c, $\chi_z = 0.824$

$$\therefore \chi_{\min} = \chi_z = 0.824$$

To calculate $k_y$

$$k_y = 1 - \frac{\mu_y N_{St}}{\chi_y A f_y} \leq 1.5$$

$$\mu_y = \bar{\lambda}_y (2 \beta_{My} - 4) + \frac{W_{p,ty} - W_{efz}}{W_{efz}} \leq 0.9$$

$$\beta_{My} = 1.8 - 0.7 \psi$$

$$\psi = 0$$

$$\therefore \beta_{My} = 1.8$$

$$\mu_y = 0.356 (2 \times 1.8 - 4) + \frac{2680 - 2370}{2370} = -0.0116$$
\[ k_y = 1 - \frac{-0.0116 \times 1552 \times 10^3}{0.945 \times 201 \times 10^2 \times 275} = 1.035 \leq 1.5 \]

\[ \frac{N_{sd}}{\chi_{min} A f_y / Y_M} + \frac{k_y M_{y, sd}}{W_{p,y} f_y / Y_M} \leq 1.0 \]

\[ \frac{1552 \times 10^3}{0.824 \times 201 \times 10^2 \times 275/1.05} + \frac{1.035 \times 299 \times 10^6}{2680 \times 10^3 \times 275/1.05} = 0.80 \leq 1.0 \]

\[ \therefore \text{ satisfactory.} \]

\[ \text{2.3.7 Lateral torsional buckling} \]

Class 1 and class 2 cross-sections should satisfy the following criterion:

\[ \frac{N_{sd}}{\chi_{x} A f_y / Y_M} + \frac{k_{LT} M_{y, sd}}{\chi_{LT} W_{p,y} f_y / Y_M} \leq 1 \]

\[ \psi = 0 \quad C_1 = 1.879 \]

\[ \lambda_{LT} = \frac{L a_{LT}}{C_1 \psi} \left[ 1 + \left( \frac{L a_{LT}}{C_1 \psi} \right)^{0.25} \right] \]

\[ = \frac{4300/84.0}{1.879 \psi} \left[ 1 + \left( \frac{4300/869}{25.66} \right)^{0.25} \right] = 31.6 \]

\[ \bar{\lambda}_{LT} = \lambda_{LT} / \lambda_1 = 31.6/86.8 = 0.364 < 0.4 \]

\[ \therefore \lambda_{LT} = 1.0 \]

To calculate \( k_{LT} \):

\[ k_{LT} = 1 - \frac{\mu_{LT} N_{sd}}{\chi_x A f_y} \leq 1 \]

where \( \mu_{LT} = 0.15 \bar{\lambda}_x \beta_{MLT} - 0.15 \leq 0.90 \)

\[ \beta_{MLT} = 1.8 - 0.7 \psi \quad \psi = 0 \]

\[ \therefore \beta_{MLT} = 1.8 \]

\[ \therefore \mu_{LT} = 0.15 \times 0.533 \times 1.8 - 0.15 = -0.0061 \]

\[ \therefore k_{LT} = 1 - \frac{-0.0061 \times 1552 \times 10^3}{0.824 \times 201 \times 10^2 \times 275} = 1.002 \]

\[ \therefore k_{LT} = 1.0 \]

\[ \frac{N_{sd}}{\chi_{x} A f_y / Y_M} + \frac{k_{LT} M_{y, sd}}{\chi_{LT} W_{p,y} f_y / Y_M} \leq 1.0 \]

\[ \frac{1552 \times 10^3}{0.824 \times 201 \times 10^2 \times 275/1.05} + \frac{1.0 \times 299 \times 10^6}{1.0 \times 2680 \times 10^3 \times 275/1.05} = 0.784 \]

\[ 0.784 < 1.0 \]
2.4 Design procedure using the concise document (C-EC3)

The two-storey frame example can also be designed using the concise version of Eurocode 3. The design is essentially the same as that in the Eurocode. However, in the following design checks the procedure in accordance with C-EC3 is simpler.

### 2.4.1 Frame imperfections

In C-EC3 the initial sway imperfection factor, \( \phi \), is obtained directly from a table:

- Number of columns, \( n_c = 3 \)
- Number of storeys, \( n_s = 2 \)

\[ \therefore \phi = \frac{1}{260} \]

**Note** This compares with 1/262 obtained using the formulae in the Eurocode.

### 2.4.2 Continuous beam

**Resistance of cross-section**

C-EC3 uses a simplified conservative expression which alleviates the need to calculate the web/section area ratio, \( a \).

From the above calculations:

\[ N_{p_{Rd}} = A f_y / Y_{Mb} = \frac{14400 \times 275}{1.05 \times 10^3} \]

\[ = 3770 \text{ kN} \]

\[ \therefore n = N_{sf}/N_{p_{Rd}} = 200/3770 = 0.0531 \]

For rolled I or H sections subjected to major axis moment:

\[ M_{N_{y,Rd}} = 1.11 M_{pfy} (1 - n) \text{ but } M_{N_{z,Rd}} \leq M_{pfy} \]

\[ M_{pfy} = W_{pfy} f_y / Y_{Mb} = \frac{3290 \times 10^3 \times 275}{1.05 \times 10^6} \]

\[ = 862 \text{ kNm} \]

\[ M_{N_{y,Rd}} = 1.11 \times 862 (1 - 0.0531) = 906 \text{ kNm} \]

\[ ie > M_{pfy} \]

\[ \therefore M_{N_{y,Rd}} = M_{pfy} = 862 \text{ kNm} \]

\[ M_{N_{y,Rd}} > M_{sd} (746 \text{ kNm}), \]

\[ \therefore \text{satisfactory.} \]

**Lateral torsional buckling**

For completeness, the lateral torsional buckling resistance of the member will be checked in accordance with the simple procedure in C-EC3.

For lateral torsional buckling:

\[ N_{sf}/N_{h,z,Rd} + M_{y_{sf}}/M_{h,Rd} \leq 1.0 \]

---

References

C-EC3 5.2.4.3

C-EC3 Table 5.3

C-EC3 5.6.1

C-EC3 Table 5.27

C-EC3 5.6.3.2
As the axial force is tension, this equation can conservatively be taken as:

\[ \frac{M_{y,\text{sd}}}{M_{b,\text{RD}}} \leq 1.0 \]

\[ M_{b,\text{RD}} = \beta_w f_b W_{p,e}/\gamma_{M1} \]

where

\[ \beta_w = 1.0 \]
\[ \gamma_{M1} = 1.05 \]
\[ W_{p,e} = 3290 \times 10^3 \text{ mm}^3 \]

\[ k = 1.0 \text{ (no fixity)} \]

\[ \psi = \frac{-746}{259} = -0.347 \]

\[ \therefore \quad (k/C_i)^{0.5} = 0.635 \]

\[ \lambda_{LT} = \frac{(k/C_i)^{0.5} \times \frac{L_h_T}{1 + \left(\frac{L_h_T}{25.66}\right)^{0.25}}}{25.66} \]

\[ = \frac{0.635 \times 2500/55.5}{1 + \left(\frac{2500/1630}{25.66}\right)^{0.25}} \]

\[ = 28.2 \]

\[ \therefore \quad f_b = 275 \text{ N/mm}^2 \]

\[ \therefore \quad M_{b,\text{RD}} = 1.0 \times 275 \times 3290 \times 10^3 \times 1.05/10^6 \]

\[ = 862 \text{ kNm} \]

\[ \therefore \quad \frac{M_{y,\text{sd}}}{M_{b,\text{RD}}} = \frac{746}{862} \]

\[ = 0.87 \]

\[ \text{i.e. } < 1.0 \]

The lateral torsional buckling resistance of the member is satisfactory in accordance with the simplified approach in C-EC3.

2.4.3 Column design (C4)
The procedures for the axially loaded beam can be applied in a similar manner to the design of the column.
Example 3
Design of a 30 m span roof truss

Introduction
This example covers the design of single-span roof trusses at 9-m centres. Lateral restraint is provided by the roof purlins. For the purpose of this example, the purlins are positioned at node points and are assumed to provide lateral restraint to the top boom at 3-m centres. In practice, this is not always possible and the design must then be adjusted accordingly. For the purposes of analysis it is assumed that all the joints in the truss are pinned.

The outline of a truss, assumed to be symmetrical about the centre line, is shown in Figure 33.

Figure 33 Outline of 30 m span truss
3.1 Truss geometry, loading and analysis

3.1.1 Variable actions
It is assumed that the roof has no access, apart from maintenance, giving a characteristic imposed load of 0.6 kN/m².

The truss will be designed for a vertical uplift of 0.55 kN/m² from wind.*

3.1.2 Permanent actions
Cladding
0.2 kN/m²

Weight of truss
0.35 kN/m²

Services*
0.05 kN/m²

Total
0.60 kN/m² (this is the characteristic permanent action*)

* It should be noted that although services are given as a permanent load in British Standard BS 6399® they need not always be present when considering wind uplift, and are ignored here in the wind reversal case.

3.1.3 Design actions
Gravity loads
It is assumed that the roof loading is applied at the nodes on the top chord of the truss. The design loads may be derived as follows:

Variable actions

\[ 0.6 \times 1.5 = 0.9 \text{ kN/m}^2 \]

Permanent actions

\[ 0.6 \times 1.35 = 0.8 \text{ kN/m}^2 \]

Total

1.7 kN/m²

The forces, F, applied to each node will be

\[ 1.7 \times 9 \times 3 = 45.9 \text{ kN}. \]

Wind uplift
It is assumed that the services loading will not be present when there is full wind causing uplift on the truss. The resulting restraining action will be the permanent action less the allowance for services

\[ 0.6 - 0.05 = 0.55 \text{ kN/m}^2 \]

This gives the following loading on the trusses:

Wind uplift

\[ 0.55 \times 3 \times 9 \times 1.5 = 22.28 \text{ kN} \]

Restraining permanent actions

\[ 0.55 \times 3 \times 9 \times 1.0 = 14.85 \text{ kN} \]

Total uplift on each node

7.43 kN
3.1.4 Member forces
A detailed method of analysis will not be given for this truss. It is assumed that this will be carried out by hand, or possibly with the help of a computer. The member forces are summarised in Table 15.

Table 15 Member forces (compression positive)

<table>
<thead>
<tr>
<th>Member</th>
<th>Force (kN) Vertical</th>
<th>Wind</th>
<th>Member</th>
<th>Force (kN) Vertical</th>
<th>Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+258</td>
<td>-42</td>
<td>11</td>
<td>-323</td>
<td>+52</td>
</tr>
<tr>
<td>2</td>
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<td>12</td>
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<td>-31</td>
</tr>
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<td>13</td>
<td>-223</td>
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</tr>
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<td>10</td>
<td>-580</td>
<td>+94</td>
<td>20</td>
<td>-11</td>
<td>+2</td>
</tr>
</tbody>
</table>
3.2 Design using angles and tees

Design assumptions
- Allow for bolted splice at mid-span.
- Assume that the effective centroids of the sections intersect at a point.

3.2.1 Top chord

Design forces compression = 581 kN
Tension caused by wind reversal = 94 kN
Length between restraints = 3.0 m on both axes

Check 191 × 229 × 37 tee grade Fe 430, cut from 457 × 191 × 74 UB.

Section properties

\begin{align*}
b &= 190.5 \text{ mm} \\
h &= 228.6 \text{ mm} \\
w &= 9.1 \text{ mm} \\
\ell &= 14.5 \text{ mm} \\
i_s &= 41.9 \text{ mm} \\
A &= 4750 \text{ mm}^2
\end{align*}

Material properties

\begin{align*}
t &\leq 40 \text{ mm} \\
f_y &= 275 \text{ N/mm}^2 \\
f_u &= 430 \text{ N/mm}^2
\end{align*}

Table 3.1

Section classification

\begin{align*}
\epsilon &= \sqrt{(235/f_y)} \\
&= \sqrt{(235/275)} = 0.924 \\
\epsilon/w &= \frac{190.5/2}{14.5} = 6.57 \\
15_\epsilon &= 15 \times 0.924 = 13.9
\end{align*}

Table 5.3.1 (Sheet 3)

\begin{align*}
h/w &= 228.6/9.1 = 25.1 \\
15_\epsilon &= 15 \times 0.924 = 13.9 < 25.1
\end{align*}

Table 5.3.1 (Sheet 4)

\begin{itemize}
- The flanges satisfy the requirements for class 3 elements.
- The stem is a class 4 element.
\end{itemize}

The section must also be checked for local as well as overall buckling.

Calculate effective section properties

\begin{align*}
\bar{b} &= \text{h for stem of tee sections} \\
&= 228.6 \text{ mm}
\end{align*}

Table 5.3.3

\begin{align*}
\psi &= \frac{\sigma_2}{\sigma_1} = 1 \\
\lambda_p^* &= \frac{\bar{b}/t}{28.4 \epsilon \sqrt{\nu}}
\end{align*}

Figure 1.1

5.3.5

5.3.5 (3)
where \( k_o = 0.43 \)

\[
\bar{\lambda}_p = \frac{228.6 / 9.1}{28.4 \times 0.924 \times \sqrt[3]{0.43}}
\]

\[
= 1.46
\]

As \( \bar{\lambda}_p > 0.673 \)

\[
\rho = \frac{\bar{\lambda}_p - 0.22}{\bar{\lambda}_p^2}
\]

\[
= \frac{1.46 - 0.22}{1.46^2}
\]

\[
= 0.582
\]

\[
b_{eff} = \rho \overline{b}
\]

\[
= 0.582 \times 228.6
\]

\[
= 133 \text{ mm}
\]

Because the flanges satisfy the requirements for a class 3 section, the full width of the flange is effective.

The effective section will be taken as formed from two rectangles, as shown in Figure 34.

\[
A_{eff} = (190.5 - 9.1) \times 14.5 + 9.1 \times 133 = 3840 \text{ mm}^2
\]

\[
\therefore \beta_A = \frac{A_{eff}}{A} = \frac{3840}{4750} = 0.808
\] 5.5.1.1 (1)

\[
z_c = \frac{[(190.5 - 9.1) \times 14.5^2 / 2 + 9.1 \times 133^2 / 2] / 3840}{25.9 \text{ mm from top of section}}
\]

**Figure 34** Effective section (shaded area not included). For design purposes this has been taken as two rectangles with no fillets (dimensions in mm)
Section in compression

\[ N_{\text{sd}} = 581 \text{ kN} \]

**Resistance of member to buckling is critical.**

It is assumed that the buckling length, \( l = \) system length between the nodes of the trusses (L = 3000 mm)

\[ l = L = 3000 \text{ mm} \]

\[ \lambda_z = \frac{l}{i_z} = \frac{3000}{41.9} = 71.6 \]

**Note** \( i_z \) is for gross section

\[ \lambda_1 = 93.9 \varepsilon = 93.9 \times 0.924 = 86.8 \]

\[ \therefore \lambda_z = \lambda_1 \beta_1^{-0.5} = 71.6 / 86.8 \times 0.808^{0.5} \]

\[ = 0.741 \]

Buckling curve c,

\[ \therefore \chi = 0.699 \]

\[ \gamma_{M1} = 1.05 \]

\[ N_{b,Rd} = \chi \beta_A \frac{L}{y_{M1}} \]

\[ = \frac{0.699 \times 0.808 \times 4750 \times 275}{1.05 \times 10^3} \]

\[ = 703 \text{ kN} \]

\[ i_e \geq N_{sd} \ (581 \text{ kN}) \]

\[ \therefore \text{satisfactory.} \]

Section in tension

The same section is used for the bottom chord, but under gravity loads it has a greater tension than the top chord has under wind uplift,

\[ \therefore \text{refer to check on bottom chord in tension.} \]

### 3.2.2 Bottom chord

*Check 191 × 229 × 37 tee grade Fe 430*

Design forces:

- Tension 580 kN
- Compression 94 kN

**Section and material properties**

Section and material properties as for top chord.

**Section in tension**

This member is designed assuming that there will be a bolted splice at the point of maximum force.

\[ N_{sd} = 580 \text{ kN} \]
Check splice
Allow for a bolted splice at midspan (see Figure 35).
Check resistance of M20 bolts, grade 4.6.

\[ A_t = 245 \, \text{mm}^2 \]
\[ f_{sb} = 400 \, \text{N/mm}^2 \]
\[ \gamma_{Mb} = 1.35 \]

\[ F_{\nu, Rd} = \frac{0.6 \cdot f_{sb} \cdot A_t}{\gamma_{Mb}} \]
\[ = \frac{0.6 \times 400 \times 245}{1.35 \times 10^3} \]
\[ = 43.6 \, \text{kN} \]
\[ d_0 = 20 + 2 = 22 \, \text{mm} \]
\[ e_1 = 40 \, \text{mm for section} = 50 \, \text{for plates} \]
\[ e_1/d_0 = 0.606 \, \text{for section} = 0.758 \, \text{for plates} \]
\[ p_1/3 \cdot d_0 - 1/4 = 0.886 \]
\[ f_{sb}/f_s = \frac{400}{430} = 0.930 (< 1.0) \]

\[ \therefore \alpha = 0.606 \, \text{for section} = 0.758 \, \text{for plates} \]

\[ F_{\nu, Rd} = 2.5 \cdot \alpha \cdot f_s \cdot d_1 \cdot t \]
\[ = \frac{2.5 \times 0.606 \times 430 \times 20 \times 9.1}{1.35 \times 10^3} \]
\[ = 87.8 \, \text{kN for stem} \]
\[ = \frac{2.5 \times 0.606 \times 430 \times 20 \times 14.5}{1.35 \times 10^3} \]
\[ = 140 \, \text{kN for flange} \]

\[ \therefore \text{for bolts in double shear:} \]
\[ F_{\nu, 3d} \leq 2 \times 43.6 = 87.2 \, \text{kN} \]

The resistance of the bolts is governed by double shear.

![Diagram of splice with bolt arrangement](image)

**Figure 35** Detail of splice showing bolting arrangement (dimensions in mm)
number of bolts required = \( N_{sd} / F_{x,Rd, max} \)

\( = 580 / 87.2 \quad = \quad 7 \) bolts

use 9 bolts to make a symmetrical detail.

To carry the shear resistance of the bolts in bearing,

\[
\text{minimum plate thickness} = \frac{F_{x,Rd}}{2.5 \alpha f_u d} \times \gamma_{Mb}
\]

\[
= \frac{43.6 \times 1.35}{2.5 \times 0.88 \times 430 \times 20 / 10^3}
\]

\( = 3.11 \text{ mm} \)

use 8 mm plates as a practical minimum.

Note: The NAD states that the bearing stress should be limited to 0.85 \((f_u + f_y) / \gamma_{Mb}\) to limit deformations. In a truss of this type this reduction should normally be applied to control the deflection.

Limiting stress \( = 0.85 \times 430 + 275 / 1.35 \quad = \quad 444 \text{ N/mm}^2 \)

Maximum bearing stress (in stem)

\[
= \frac{580}{9} \times \frac{10^3}{20 \times 9.1}
\]

\( \leq 444 \text{ N/mm}^2 \)

Check tension resistance of member

Two checks are required for the tension resistance of the member:

- the resistance of the whole section \((N_{p,Rd})\), and
- the resistance of the section where bolts occur \((N_{u,Rd})\).

Resistance of the whole section

\[
N_{p,Rd} = \frac{A f_u \gamma_{M0}}{\gamma_{M2}}
\]

\[
= \frac{4750 \times 275}{1.05 \times 10^3}
\]

\( = 1244 \text{ kN} \) 5.4.3 (1)

Resistance at bolt holes

Allow for three 22-mm diameter holes at any section.

\[
A_{net} = A \text{ minus area of all bolt holes}
\]

\( A_{net} = 4750 - 22 \times (9.1 + 2 \times 14.5) = 3910 \text{ mm}^2 \)

\[
\gamma_{M2} = 1.20
\]

\[
N_{u,Rd} = 0.9 A_{net} f_y \gamma_{M2}
\]

\[
= \frac{0.9 \times 3910 \times 430}{1.20 \times 10^3}
\]

\( = 1261 \text{ kN} \) 5.4.3 (1)

\( \therefore \) \( N_{u,Rd} \) is the smaller of \( N_{p,Rd} \) and \( N_{u,Rd} = 1244 \text{ kN} \)

\( \geq N_{sd} (580 \text{ kN}) \)

\( \therefore \) satisfactory.
Section in compression

\[ N_{sd} = 94 \text{ kN} \]

Resistance of member to buckling is critical.

Calculate spacing of lateral restraints.

Let \[ N_{b, Rd} = N_{sd} \]
\[ N_{b, Rd} = \chi \beta_A A f_y / \gamma_M \]
\[ \therefore \chi = \frac{N_{b, Rd} \gamma_M}{\beta_A A f_y} \]
\[ \geq \frac{94 \times 1.05}{0.808 \times 4750 \times 275/10^3} \]
\[ \geq 0.094 \]

From buckling curve c, the value of \( \lambda \) to give this value of \( \chi \) is 3.01.

\[ \lambda_z = \frac{\lambda_{y_1}}{\beta_A^{0.5}} \]
\[ \therefore \lambda_z = \frac{3.01 \times 86.8}{0.808^{0.5}} \]
\[ \leq 290 \]

Note: The NAD requires \( \lambda_z \) for a member normally a tie to be less than 350.

\[ l = \lambda_z l_z \]
\[ \therefore l \leq 290 \times 41.9 \]
\[ \leq 12151 \text{ mm} \]

\[ \therefore \] a minimum of two positions of intermediate lateral restraint are required to the bottom chord to leave unrestrained length \( \leq 12.15 \).

Check stem splice plates

Area of flange = 190.5 \times 14.5
= 2760.2 \text{ mm}^2

Area of tee = 4750 \text{ mm}^2

\[ \therefore \text{area of stem of tee} = 4750 - 2760.2 \]
\[ = 1988 \text{ mm}^2 \]

Force in stem of tee = \( \frac{1988}{4750} \times 580 \)
= 243 \text{ kN}

Net area of stem splice plates = 2 \times 8 \times (175 - 22)
= 2448 \text{ mm}^2

Resistance of stem splice plates = 2448 \times 275/1.05/10^3
= 641 \text{ kN}

641 > 243
\[ \therefore \text{satisfactory.} \]

Resistance of stem bolts = 3 \times 87.2 = 262 \text{ kN}
262 > 243
\[ \therefore \text{satisfactory.} \]
Check flange splice plates

\[ \text{Force in flange} = 580 - 243 = 337 \text{ kN} \]

\[ \text{Net area of flange splice plates} = 2 \times 8 \times (70 - 22) + 8 \times (180 - 44) = 1856 \text{ mm}^2 \]

\[ \text{Resistance of flange splice plates} = 1856 \times 275/1.05/10^3 = 486 \text{ kN} \]

\[ 486 \gg 337 \]
\[ \therefore \text{ satisfactory.} \]

\[ \text{Resistance of flange bolts} = 6 \times 87.2 = 523 \text{ kN} \]

\[ 523 \gg 337 \]
\[ \therefore \text{ satisfactory.} \]

3.2.3 Diagonal brace members

The critical member is that adjacent to the support (number 11).

Design forces
- Tension \(323\) kN
- Compression \(52\) kN

Check \(120 \times 120 \times 10\) angle grade Fe 430.

**Section properties**
- \(h = 120 \text{ mm}\)
- \(t = 10 \text{ mm}\)
- \(i_y = 36.7 \text{ mm}\)
- \(A = 2320 \text{ mm}^2\)

**Material properties**
- \(t \leq 40 \text{ mm}\)
- \(f_y = 275 \text{ N/mm}^2\)
- \(f_u = 430 \text{ mm}^2\)

**Section in tension**
- \(N_{sd} = 323 \text{ kN}\)

Welded end connections:

Effective area = total area of angle with equal legs in tension.

\[ N_{t,\text{rd}} = N_{p/\text{rd}} = \frac{A_{\text{eff}} f_y}{f_{\text{Mlt}}} \]

\[ = \frac{2320 \times 275}{1.05 \times 10^3} = 608 \text{ kN} \]

\[ \geq N_{sd} (323 \text{ kN}) \]
\[ \therefore \text{ satisfactory.} \]

**Section in compression**
- \(N_{sd} = 52 \text{ kN}\)
Section classification

\[ \varepsilon = \sqrt{\frac{235}{f_y}} \]
\[ = \sqrt{\frac{235}{275}} = 0.924 \]
\[ \frac{h}{t} = \frac{120}{10} = 12.0 \]
\[ 11.5 \varepsilon = 11.5 \times 0.924 = 10.6 < 12.0 \]

\[ \therefore \text{ section is class 4.} \]

Calculate effective section properties

\[ \bar{b} = h = 120 \text{ mm} \]

Uniform compression

\[ \therefore \psi = \frac{\sigma_y}{\sigma_1} = 1 \]
\[ \therefore k_\sigma = 0.43 \]

\[ \therefore \bar{\lambda}_p = \frac{\bar{b}/t}{28.4 \varepsilon \times \sqrt{k_\sigma}} \]
\[ = \frac{120/10}{28.4 \times 0.924 \times \sqrt{0.43}} = 0.697 \]

For \( \bar{\lambda}_p > 0.673 \), the value of \( \rho = \frac{\bar{\lambda}_p - 0.22}{\lambda_p^2} \)
\[ = \frac{0.697 - 0.22}{0.697^2} = 0.982 \]

\[ \therefore b_{\text{eff}} = \rho \bar{b} = 0.982 \times 120 = 118 \text{ mm} \]

Figure 36 shows the effective section of the angle.

![Figure 36](image)

**Figure 36** Effective section (dimensions in mm). The shaded areas are not included in the effective section

\[ A_{\text{eff}} = 2320 - 2 \times 11 \times 2 = 2276 \text{ mm}^2 \]

\[ \beta_A = \frac{A_{\text{eff}}}{A} = \frac{2276}{2320} = 0.981 \]

Considering the angle as two rectangular plates:

\[ z_c = \frac{10 \times 118^{1/2} + 108 \times 10^{1/2}}{10 \times 118 + 108 \times 10} \]
\[ = 33.2 \text{ mm} \]
Resistance of member to buckling is critical.

\[ l = L = (3000^2 + 2250)^{0.5} = 3750 \text{ mm} \]

\[ \lambda_v = \frac{l}{i_v} = \frac{3750}{23.6} = 159 (\leq 350) \]

\[ \lambda_z = \frac{l}{i_z} = \frac{3750}{36.7} = 102 \]

\[ \lambda_1 = 93.9 \varepsilon = 93.9 \times 0.924 = 86.8 \]

\[ \bar{\lambda}_v = \frac{\lambda_v}{\lambda_1} \beta_A^{0.5} = \frac{159/86.8 \times 0.982^{0.5}}{1.16} = 1.82 \]

\[ \bar{\lambda}_z = 0.35 + 0.7 \bar{\lambda}_v = 0.35 + 0.7 \times 1.82 = 1.62 \]

\[ \bar{\lambda}_{eff,v} = 0.5 + 0.7 \bar{\lambda}_v = 0.5 + 0.7 \times 1.16 = 1.3 \]

Buckling curve c, Table 5.5.3

\[ : \chi_{min} = \chi_v = 0.279 \]

\[ : N_{bd,rd} = \chi \beta_A A f_y / \gamma_M \]

\[ = \frac{0.279 \times 0.982 \times 2320 \times 275}{1.05 \times 10^3} \]

\[ = 166 \text{ kN} \]

\[ \geq N_{sd} (52 \text{ kN}) \]

\[ : \text{satisfactory.} \]

3.2.4 Vertical brace members

Member 12 is the critical section with:

Maximum compression = 194 kN
Maximum tension = 31 kN

Check 120 \times 120 \times 10 angle grade Fe 430.

Section and material properties

Section and material properties as for diagonal brace members.

Section in compression

\[ N_{sd} = 194 \text{ kN} \]

Resistance of member to buckling is critical.

The buckling length is taken as the system length

\[ l = L (2400 \text{ mm}) \]

\[ \lambda_v = \frac{l}{i_v} = 2400/23.6 = 102 (\leq 180) \]

\[ \lambda_z = \frac{l}{i_z} = 2400/36.7 = 65.4 \]

\[ \bar{\lambda}_v = \frac{\lambda_v}{\lambda_1} \beta_A^{0.5} = \frac{102/86.8 \times 0.982^{0.5}}{1.16} = 1.16 \]

\[ \bar{\lambda}_z = \frac{\lambda_v}{\lambda_1} \beta_A^{0.5} = \frac{65.4/86.8 \times 0.982^{0.5}}{0.747} \]

References

5.5.1

5.5.2

5.5.3

5.5.1.4

NAD 6.1.3 d)

5.5.1.2 (1)

5.5.1.4 (3)

Table 5.5.2

5.5.1.1

103
\[ \lambda_{x_{\text{eff}}} = 0.35 + 0.7 \lambda_v = 0.35 + 0.7 \times 1.16 = 1.16 \]
\[ \lambda_{x_{\text{eff}}} = 0.5 + 0.7 \lambda_z = 0.5 + 0.7 \times 0.747 = 1.023 \]
\[ \therefore \chi_{\text{min}} = \chi_v = 0.453 \text{ (curve c)} \]
\[ \therefore N_{b,\text{rd}} = \chi \beta_A A f_i / \gamma_M = \frac{0.453 \times 0.982 \times 2320 \times 275}{1.05 \times 10^3} \]
\[ = 270 \text{ kN} \]
\[ \therefore \text{satisfactory in compression and, by inspection, in tension.} \]

3.2.5 Design of connection between members 1, 11 and the support

![Diagram of connection](image)

Figure 37 Elevation of end connection (dimensions in mm)

Force on angle = 323 kN

The connection shown in Figure 37 is to be designed as typical of all the truss connections. In a full design a similar check would be required at every connection in the truss.
To reduce the effects of eccentricity of the welds on the angles, the centroid of the weld group will be located as near as possible to the centre line of the members. (This is the designer’s decision, it is not a Eurocode rule.)

For a 120 × 120 × 10 angle this is 33.1 mm from the back. A close approximation is to make the weld on the back corner four times that on the toe of the angle.

Assume 8 mm weld:

\[ f_{vw} a = \frac{f_y \sqrt{3}}{\beta_w y_{Mw}} = \frac{275\sqrt{3}}{0.85 \times 1.35} = 138 \text{ N/mm}^2 \]

\[ \beta_w = 0.85 \text{ for grade Fe 430 steel} \]

With 8 mm weld throat thickness \( = \frac{8}{\sqrt{2}} = 5.66 \text{ mm} \)

\[ F_{vw} = A f_{vw} a = 5.66 \times 138 = 781 \text{ N/mm run} \]

Length of weld required \( = \frac{323}{781 \times 10^{-3}} = 414 \text{ mm} \).

Length of weld provided \( = 120 + 241 + 187 + 91 \)
\( = 639 \text{ mm} \)

This is more than adequate.
\[ \therefore \text{ satisfactory.} \]

### 3.2.6 Deflection check

The deflection of a truss with non-parallel booms may be found by hand calculations (using a unit load), graphically (using a Williot-Mohr diagram) or by means of a computer analysis.

In this case the deflections were found to be as follows:

- With imposed load \[ \delta_{max} = 39 \text{ mm} \]
  \[ \delta_2 = 19 \text{ mm} \]
- With wind load \[ \delta_{max} = 0 \text{ mm} \]
  \[ \delta_2 = -18 \text{ mm} \]

The limiting deflections given in Eurocode 3 for a structure not supporting a brittle finish are:

\[ \delta_{max} \leq \frac{L}{200} = \frac{3000}{200} = 150 \text{ mm} \]

\[ \delta_2 \leq \frac{L}{250} = \frac{3000}{250} = 120 \text{ mm} \]

The calculated deflections fall within these limits,
\[ \therefore \text{ satisfactory.} \]

A roof with a fall as steep as this (more than 5%) will not be prone to ponding.

4.2.3

References

4.2.2
3.3 **Design using circular hollow sections**
The members of the previous roof truss are here determined assuming that they are circular hollow sections.

### 3.3.1 Analysis
For trusses composed of CHS members, it is usually more efficient to use gap joints (see Annex K, Figure K.1).

Assume
- $193.7 \times 8$ CHS grade 43C for chord members,
- $114.3 \times 5.0$ CHS grade 43C for diagonal members, and
- $88.9 \times 5.0$ CHS grade 43C for vertical members.

In a design using these members, check whether eccentricities can be ignored and a simple pin-jointed analysis used.

However, for all hollow section brace members (internal members, such as diagonals and verticals), the joint resistances must be checked as part of the member selection process, as well as the basic member resistances.

**Check eccentricities of joints**

*Support joint members I and II*
The centroids of the members may be arranged to pass through the welding of the chord to the plate. This means that eccentricity need not be considered.
Figure 38 shows support details for members 1, 11 and 6, while Figure 39 shows the top chord joint between members 1, 2, 12 and 13.

Figure 38 Support details, members 1, 11 and 6 (dimensions in mm)
Figure 39  Top chord joint between members 1, 2, 12 and 13 (dimensions in mm)

The approximate dimensions indicate an eccentricity of +24 mm at this joint. This is based on a gap of 12 mm, which is greater than $t_1 + t_2$
where $t_1$ and $t_2$ are the wall thicknesses of the internal members of the trusses (50 mm), giving a minimum gap of 10 mm.

The moment from the eccentricities may be neglected if:

$$-0.55 d_o \leq e \leq 0.25 d_o$$

In this instance $e/d_o = 24/193.7 = +0.12$

$\therefore$ the eccentricity at this node can be neglected.
The top chord joint between members 4, 5, 18 and 19 (Figure 40) is considered in the same way as the previous one, except that, because of a change of slope, \( c = 39 \text{ mm} \)

\[
\therefore \quad \frac{c}{d_o} = \frac{39}{193.7} = +0.20
\]

\[
\therefore \quad \text{this is within the limits set for neglecting the eccentricity.}
\]

The joints on the top chord will have eccentricities that vary between 35 mm and 42 mm, and are within the limits given in clause K.3.

![Figure 40](https://example.com/figure40.png)

**Figure 40** Top chord joint between members 4, 5, 18 and 19 (dimensions in mm)

The joints on the bottom chord are similar to those shown in Figure 41, and satisfy the eccentricity requirements for a pin-jointed truss.

![Figure 41](https://example.com/figure41.png)

**Figure 41** Connections between members 6, 7, 11 and 12 (dimensions in mm)
Check joint geometry

\[
\frac{d_i}{d_o} = \frac{114.2}{193.7} = 0.59
\]

88.9/193.7 = 0.46
\[\geq 0.2\]
\[\leq 1.0\]

\[
\frac{d_i}{2t_i} = \frac{114.3}{2 \times 5} = 11.4
\]

\[
\frac{88.9}{2 \times 5} = 8.9
\]
\[\leq 25\]
\[\geq 5\]

\[
\frac{d_i}{2t_o} = \frac{193.7}{2 \times 8} = 12.1
\]
\[\leq 25\]
\[\geq 5\]

\[
\lambda_{ow} = \frac{q}{p} \times 100 = \frac{110}{190} \times 100 = 58\%
\]
\[\geq 25\%\]

The diagonal internal members have angles between 37.8° and 47.4° (44.5 + 2.9). The geometry of the joints is therefore within the range of validity given in Table K.6.1, so Table K.6.2 can be used for calculating design resistance of joints.

Check member stiffness coefficients

System length/member depth for:

- Chord members = 3002/193.7 = 15.5 (> 12)
- Diagonal members = 3670/114.3 = 32 (> 24)
- Vertical members = 2400/88.9 = 27 (> 24)

:. satisfactory.

3.3.2 Top chord

Maximum forces:

Tension = 94 kN
Compression = 581 kN

Check 193.7 x 8.0 CHS grade 43C.

Section properties

- \(d = 193.7\ mm\)
- \(t = 8.0\ mm\)
- \(i = 65.7\ mm\)
- \(A = 4670\ mm^2\)
Material properties

\[ t \leq 40 \text{ mm} \]

\[ \therefore \, f_y = 275 \text{ N/mm}^2 \]

\[ f_u = 430 \text{ N/mm}^2 \]

Section classification

\[ \varepsilon = \sqrt{\frac{235}{f_y}} \]

\[ = \sqrt{\frac{235}{275}} = 0.924 \]

\[ d/t = 193.7/8 = 24.2 \]

\[ 50 \varepsilon^2 = 50 \times (0.924)^2 = 42.7 \]

\[ \therefore \text{ section is class 1.} \]

Section in compression

Resistance of cross-section to axial force

\[ N_{p,Rd} = A \frac{f_y}{f_{M0}} \]

\[ = \frac{4670 \times 275}{1.05 \times 10^3} = 1223 \text{ kN} \]

Resistance of member to buckling

\[ \ell = L = 3000 \text{ mm} \]

\[ \lambda = \frac{L}{h} = 3000/65.7 = 45.7 \ (\leq 180) \]

\[ \lambda_t = 93.9 \varepsilon = 93.9 \times 0.924 = 86.8 \]

\[ \lambda_\lambda = \frac{\ell}{\lambda_t \beta_{\lambda}^{0.5}} = 45.7/86.8 \times 1^{0.5} = 0.53 \]

Buckling curve a

\[ \chi = 0.914 \]

The compression resistance

\[ N_{v,Rd} = \chi \beta_{\lambda} A \frac{f_y}{f_{M0}} \]

\[ = \frac{0.914 \times 1 \times 4760 \times 275}{1.05 \times 10^3} \]

\[ = 1140 \text{ kN} \]

\[ \therefore \text{ satisfactory in compression and, by inspection, in tension due to wind reversal.} \]

3.3.3 Bottom chord

Maximum forces:

Tension = 580 kN

Compression = 94 kN

Check 193.7 \times 8.0 CHS grade 43C

Section and material properties

Section and material properties as for top chord.
Section in tension

\[ N_{sd} = 580 \text{ kN} \]

Resistance of cross-section to axial force

The resistance of the cross-section is given in the design of the top chord as 1223 kN. This is greater than the tension, so the section is adequate in tension.

\[ \therefore \text{ satisfactory.} \]

Member in compression

\[ N_{sc} = 94 \text{ kN} \]

Resistance of the member to buckling is critical.

Assume that a brace will be provided at the centre of the truss, dividing the chord in two and giving a length of 15 000 mm between restraints.

\[ \lambda = \frac{15 000}{65.7} = 228 \text{ (the NAD limits this to 350)} \]

\[ \bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{228}{86.8} = 2.6 \]

Using curve a from Table 5.5.2, \[ \chi = 0.136 \]

Buckling resistance \[ N_{b,Rd} = \frac{0.136 \times 1 \times 4670 \times 275}{1.05 \times 10^3} \]

\[ = 166 \text{ kN} \]

This is greater than the compression \[ N_{sc} (= 94 \text{ kN}) \].

Provide a single line of bracing on the centre of the truss.

Note This is less than required for the tee-section tension members.

---

3.3.4 Diagonal brace member (based on member 11)

Maximum forces:

- Tension \[ 323 \text{ kN} \]
- Compression \[ 52 \text{ kN} \]

Check \( 114.3 \times 5.0 \) CHS grade 43C.

Section properties

\[ \begin{align*}
&d = 114.3 \text{ mm} \\
&t = 5.0 \text{ mm} \\
&i = 38.7 \text{ mm} \\
&A = 1720 \text{ mm}^2 \\
&W_{pR} = 59.8 \times 10^3 \text{ mm}^3 \\
&W_{cr} = 45.0 \times 10^3 \text{ mm}^3 \\
\end{align*} \]

Material properties

\[ \begin{align*}
&t \leq 40 \text{ mm} \\
&\therefore f_y = 275 \text{ N/mm}^2 \\
&f_u = 430 \text{ N/mm}^2 \\
\end{align*} \]
Section classification

\[ \frac{d}{t} = \frac{114.3}{5.0} = 22.9 \]

\[ 0 \leq \frac{t}{c} = 50 \times (0.924)^2 = 42.7 \geq 22.9 \]

\[ \therefore \text{section is class 1} \]

Section in tension

Resistance of cross-section to axial force

\[ N_{p/kd} = \frac{Af_y}{\gamma_{M0}} \]

\[ = \frac{1720 \times 275}{1.05 \times 10^3} = 450 \text{ kN} \]

\[ 450 \text{ kN} > 323 \text{ kN} \]

\[ \therefore \text{satisfactory.} \]

For member 11:

Check chord plastification of N joint

\[ N_{1,kp} = \frac{f_{ye} t_i^2}{\sin \theta_i} \times (1.8 + 10.2 \frac{d}{d_o}) k_p k_g (1.1/\gamma_{M0}) \]

\[ f_{ye} = \text{chord yield strength} = 275 \text{ N/mm}^2 \]

\[ t_o = \text{thickness of chord wall} = 8 \text{ mm} \]

\[ k_p = 1.0 \text{ as the member is in tension} \]

\[ \gamma = \frac{d}{t_o} = \frac{193.7}{2 \times 8} = 12.1 \]

\[ k_g = 2.2 \text{ (taken from Figure K.3)} \]

This gives:

\[ N_{1,kp} = \frac{275 \times 8^2}{\sin 37.8} \times [1.8 + 10.2 \times (114.3/193.7)] \times 1.0 \times 2.2 \times (1.1/1.05) \times 10^{-3} \]

\[ = 517 \text{ kN} \]

\[ 517 \text{ kN} > 323 \text{ kN} \]

\[ \therefore \text{chord plastification is satisfactory.} \]

Check punching shear

\[ N_{1,kp} = \frac{f_{ye}}{\sqrt{3}} \pi d_i \frac{1 + \sin \theta_i}{2 \sin^2 \theta_i} \frac{1.1}{\gamma_{M0}} \]

\[ = \frac{275}{\sqrt{3}} \times 5 \times \pi \times 114.3 \times \frac{1 + \sin 37.8}{2 \times \sin^2 37.8} \times \frac{1.1}{1.05 \times 10^3} \]

\[ = 641 \text{ kN} > 323 \text{ kN} \]

\[ \therefore \text{punching shear is satisfactory.} \]
Joint efficiency
The joint efficiency depends on the lesser of the chord plastification and the punching shear values.

\[ N_{p/Rd} = 450 \text{ kN} \]

The resistance to chord plastification (517 kN) is greater than \( N_{p/Rd} \) (450 kN), therefore chord efficiency is 100%.

Section in compression
Resistance of member to buckling is critical.

The buckling length may be taken as 0.9 \( L \), where \( L \) is the system length of the member.

\[
\lambda = \frac{l}{i} = \frac{3375/38.7}{87 (\leq 180)}
\]

\[
\lambda_1 = 93.9 \varepsilon = 93.9 \times 0.924 = 86.8
\]

\[
\beta_\lambda = 1
\]

\[
\bar{\lambda} = \lambda_1 \beta_\lambda^{0.5} = 87/86.8 \times 1^{0.5} = 1.0
\]

Buckling curve a

\[
\therefore \chi = 0.666
\]

\[
N_{b/Rd} = \frac{0.666 \times 1 \times 1720 \times 275}{1.05 \times 10^3}
\]

\[
= 300 \text{ kN} > 52 \text{ kN}
\]

\[
\therefore \text{ satisfactory.}
\]

3.3.5 Vertical brace members (based on member 12)

Forces:

<table>
<thead>
<tr>
<th></th>
<th>Tension</th>
<th>31 kN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Compression</td>
<td>194 kN</td>
</tr>
</tbody>
</table>

Check \( 88.9 \times 5 \) CHS grade 43C.

\[
d = 88.9 \text{ mm}
\]

\[
t = 5.0 \text{ mm}
\]

\[
i = 29.7 \text{ mm}
\]

\[
A = 1320 \text{ mm}^2
\]

Material properties: same as those of other sections.

Local buckling

\[
d/t = 88.9/5.0 = 18
\]

Limit, as before \( = 42.7 \)

So the section is class 1,

\[
\therefore \text{ satisfactory.}
\]
Resistance of section to axial force

\[ \lambda = 0.9 \times 2400/29.7 = 73 \]
\[ \overline{\lambda} = 73/86.8 = 0.84 \]

From Table 5.5.3 curve a,

\[ \chi = 0.772 \]
\[ N_{b,Rd} = \frac{0.772 \times 1 \times 1320 \times 275}{1.05 \times 10^3} = 267 \text{ kN} \]

This is greater than the force \( N_{rd} \) (194 kN).

\[ \therefore \text{ satisfactory in compression and, by inspection, in tension.} \]

For member 12:

Check chord plastification of N joint

\[ N_{\lambda,Rd} = \frac{f_{\mu}^2 t_0}{\sin \theta_2} \times (1.8 + 10.2 \frac{d}{d_o}) k_p k_z (1.1/\chi_0) \]

\[ n_p = \sigma_p / f_{\mu o} = (194 \times 10^3/1320)/275 = 0.53 \]
\[ k_p = 0.75 \]

\[ N_{\lambda,Rd} = \frac{275 \times 8^2}{\sin 90} \times [1.8 + 10.2 \times (88.9/193.7)] \times 0.75 \times 2.2 (1.1/1.05) \times 10^{-3} \]
\[ = 197 \text{ kN} \]

197 kN > 194 kN

\[ \therefore \text{ chord plastification is satisfactory.} \]

Check punching shear

\[ N_{\lambda,Rd} = \frac{f_{\mu}^2 t_0 \pi d_o}{\sqrt{3} \times \frac{1 + \sin \theta_2}{2 \sin^2 \theta_2} \times \frac{1.1}{\gamma_{M_j}}} \]

\[ = \frac{275}{\sqrt{3}} \times 8 \times \pi \times 88.9 \times \frac{1 + \sin 90}{2 \times \sin^2 90} \times \frac{1.1}{1.05 \times 10^3} \]
\[ = 371.6 \text{ kN} > 194 \text{ kN} \]

\[ \therefore \text{ punching shear is satisfactory.} \]

Joint efficiency

\[ N_{p,Rd} = 1320 \times 275/1.05 \times 10^3 = 363 \text{ kN} \]

The resistance to chord plastification is less than the punching shear resistance,

\[ \therefore \text{ efficiency of the joint} = \frac{197}{363} = 54.3\% \]

Note All the above resistances are satisfactory, but a more efficient joint would be obtained by using a thinner CHS of larger diameter.
3.3.6 Connections
The connections between members 6, 7, 11 and 12 are studied here as an example (see Figure 41 repeated here for easy reference). The designer should check all the connections in turn, beginning with those carrying the greatest forces.

![Diagram of connections between members 6, 7, 11, and 12 with forces and dimensions labeled.](image)

Figure 41 (repeated) Connections between members 6, 7, 11 and 12 (dimensions in mm)

Welding
In this type of joint the welding is partly fillet and partly butt, depending on the geometry at any particular point. The weld is required to satisfy clause K.5 of the Eurocode. For a simple calculation, clause K.5(4) gives $a/t \geq 0.87\alpha$. Alternatively, clause K.5(5) permits a smaller weld, so that the actual force and the joint efficiency can cope with non-uniform distributions of load. The joint efficiency depends on the lesser of the chord plastification and the punching shear values.

Member 11 114.3 $\times$ 5 CHS 323 kN tension
Member 12 88.9 $\times$ 5 CHS 194 kN compression

Member 11
For a fully loaded member:

\[
a/t \geq 0.87\alpha
\]

\[
\alpha = \frac{1.1}{\gamma_{Mf}} \times \frac{f_{y}}{1.25} = \frac{1.1}{1.05} = 1.35 \times 1.25 = 1.13
\]

\[
a/t \geq 0.87 \times 1.13 = 0.98
\]

\[
a \geq 0.98 \times 5 = 4.9 \text{ mm}
\]

But applied load (323 kN) is less than $N_{y,\text{Sd}}$ (450 kN), therefore a weld size smaller than 4.9 mm can be used, depending on the joint efficiency.

From Section 3.3.4 joint efficiency is 100%,

\[
\therefore a/t \geq 0.98 \times 1 \times \frac{323}{450} = 0.71
\]

\[
a = 0.71 \times 5 = 3.55
\]

Use a weld with a throat thickness $\geq 4$ mm all round.
Member 12
For a fully loaded member:
\[ a/t \geq 0.98 \]
\[ N_{p,ld} = 363 \text{ kN} \]
Joint efficiency = 54.3% (see Section 3.3.5)
\[ \therefore \text{minimum weld size} = t \times 0.98 \times \frac{100}{54.3} \times \frac{194}{363} = 0.96 \ t \]
\[ = 0.96 \times 5 = 4.8 \text{ mm} \]
\[ \therefore \text{use a weld with 5 mm throat thickness all round.} \]

3.3.7 Deflection check
The deflections may be determined in the same way as for the tee and angle truss. In this case the maximum deflection is 71 mm = span/422. This is adequate to meet the requirements of Eurocode 3.

It should be noted that, because of the smaller areas of the members, the deflection of the hollow section truss will be higher than that of the truss formed of angles and tees. Furthermore, if the trusses are bolted, the movement in the joints would increase the deflection above the calculated value.
Example 4
Design of a gantry girder to support a
100 kN capacity crane

4.1 Girder geometry, loading and analysis
Design a simply supported gantry girder spanning 8.0 m, using a universal beam and top flange plate in grade Fe 430 steel. It is to carry an overhead travelling crane for medium and heavy workshop duty. The crane details are shown in Figure 42.

![Diagram of gantry girder and crane](image)

Figure 42 Details of 100 kN capacity crane (dimensions in mm)

<table>
<thead>
<tr>
<th>Crane capacity</th>
<th>$W_{cp}$ = 100 kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of crab</td>
<td>$W_{cb}$ = 20 kN</td>
</tr>
<tr>
<td>Weight of crane bridge</td>
<td>$W_c$ = 80 kN</td>
</tr>
<tr>
<td>Span of crane bridge</td>
<td>$L_c$ = 15 m</td>
</tr>
<tr>
<td>Wheel spacing in end carriage</td>
<td>$a_w$ = 4.0 m</td>
</tr>
<tr>
<td>Minimum hook approach</td>
<td>$a_h$ = 1.0 m</td>
</tr>
</tbody>
</table>

The gantry girder details are:
- Weight (approximate) $W_G$ = 15 kN
- Span $L = 8.0$ m

Note The design parameters have been chosen to be the same as those used in the gantry girder example in the SCI's Steelwork design guide to BS 5950: Part 1: 1990. Volume 2: Worked examples. This permits an effective comparison to be drawn between the British Standard and the Eurocode design procedures. However, it should be noted that the end carriage dimensions are not particularly representative for a crane with this level of loading and span.

4.1.1 Calculation of loads

Load factors
Loads are to be combined in the following ways and the most unfavourable value used in design:

- Consider only the most unfavourable variable action
  $$\sum_{j} \gamma_{Q_j} G_{k_j} + \gamma_{Q_k} Q_{k_i}$$

- Consider all unfavourable variable actions
  $$\sum_{j} \gamma_{Q_j} G_{k_j} + 0.9 \sum_{i=1}^{n} \gamma_{Q_i} Q_{k_i}$$

References
Partial safety factors

\[ \gamma_{Gd} = 1.35 \]
\[ \gamma_{O,1} = 1.50 \]

Pending the publication of Eurocode 3: Part 6, Eurocode 1 or a CEN crane standard, Table 3 of the UK National Application Document for Eurocode 3: Part 1.1 recommends that the resistance of a crane girder and its supports be checked for the following load cases:

- Vertical load (see Section 4.3.1)
- Horizontal load (see Section 4.3.2)
- \( 0.9 \times (\text{vertical load} + \text{horizontal load}) \) (see Section 4.3.4)

**Vertical loading**

Variable actions:

Maximum characteristic static wheel load, \( W_{us} \)

\[
W_{us} = \frac{1}{2} \left[ \frac{W_{c}}{2} + (W_{cb} + W_{cap}) \left( \frac{L_{c} - a_{b}}{L_{c}} \right) \right]
\]

\[
= \frac{1}{2} \left[ 40 + (20 + 100) \times \frac{14}{15} \right] = 76 \text{ kN}
\]

Maximum design static wheel load, \( W_{w} \)

\[ \gamma_{O,1} W_{us} = 1.50 \times 76 = 114 \text{ kN} \]

The crane is classified as load class Q3, therefore an impact factor of 1.30 must be applied to the static wheel load to allow for shock loading\(^{15}\).

Maximum design wheel load allowing for impact:

\[ W_{v} = 1.3 W_{w} = 1.3 \times 114 = 148.2 \text{ kN} \]

Permanent actions

Design value of girder self-weight, \( W_{G}^{'} \)

\[ \gamma_{Gd} W_{G} = 1.35 \times 15 = 20.3 \text{ kN} \]

**Horizontal loading**

As the crane is class Q3, crabbing action must be taken into account\(^{13}\). However, the couple caused by crabbing action should not be combined with the horizontal forces caused by surge.

**Variable actions caused by surge**

Transverse loads: assume a force equal to 10% of the combined weight of the crab and the load lifted\(^{6}\).

Maximum horizontal transverse load per wheel

\[
= \frac{0.1 (W_{cb} + W_{cap})}{\text{number of wheels}}
\]

\[= \frac{0.1 (20 + 100)}{4} = 3 \text{ kN} \]

Design value per wheel, \( W_{H} \)

\[ W_{H} = 1.5 \times 3 = 4.5 \text{ kN} \]
Longitudinal loads: assume a force equal to 5% of the static wheel loads.

Maximum longitudinal load per wheel \( = 0.05 \times 76 = 3.8 \text{ kN} \)

Design value, \( W_{H2} = 1.5 \times 3.8 = 5.7 \text{ kN} \)

**Variable actions caused by crabbing**

Crabbing force transverse to the rail, per wheel,

\[
F_R = \frac{L \cdot W_w}{40 \cdot a_w} \geq \frac{W_w}{20}
\]

\[
\frac{L \cdot W_w}{40 \cdot a_w} = \frac{15 \times 114}{40 \times 4} = 10.7 \text{ kN}
\]

\[
\frac{W_w}{20} = \frac{114}{20} = 5.7 \text{ kN}
\]

\[
\therefore F_R = 10.7 \text{ kN}
\]

### 4.1.2 Design internal moments and forces

**Due to vertical load**

The worst case for bending moment caused by crane wheel loads is shown in Figure 43.

![Diagram of Girder](image)

**Figure 43** Worst case for bending moment caused by crane wheel loads (dimensions in mm)

Maximum moment, \( M_v = \frac{2}{8} \frac{W_v}{L} \left( \frac{L}{2} - \frac{a_w}{4} \right) + W_c \frac{L}{8} \)

\[
= \frac{2 \times 148.2}{8} \left( \frac{8}{2} - \frac{4}{4} \right) + 20.3 \times \frac{8}{8}
\]

\[
= 353.75 \text{ kNm}
\]
The maximum shear force occurs when one wheel is virtually over a support, as shown in Figure 44.

![Diagram showing shear forces and reactions](image)

**Figure 44** Worst case for vertical shear (dimensions in mm)

\[
F_{v,\text{max}} = \text{maximum reaction at A}
\]

\[
= W_v + W_c \times \frac{L - a_w}{L} + \frac{W_G^{'}}{2}
\]

\[
= 148.2 + 148.2 \times \frac{8 - 4}{8} + \frac{20.3}{2} = 232.5 \text{ kN}
\]

**Due to horizontal loading**

*Due to surge (Figure 45)*

![Diagram showing horizontal forces](image)

**Figure 45** Design horizontal bending moments due to surge (dimensions in mm)

Maximum moment, \( M_{H1} = \frac{2 W_{H1} (L - a_w^2)}{L} \)

\[
= 2 \times 4.5 \times \frac{8}{8} \left( \frac{8 - 4}{2} \right)^2
\]

\[
= 10.1 \text{ kNm}
\]

*Due to crabbing (Figure 46)*

![Diagram showing forces due to crabbing](image)

**Figure 46** Design horizontal bending moments due to crabbing
Maximum horizontal moment due to crabbing, $M_R$

= $F_R \times L/4$ (only for this case)

= $10.69 \times 8/4$  = 21.4 kNm

When one wheel is over a support, maximum moment, $M_R$

= $W \times (a - a^2/L)$

Maximum horizontal design moment $M_H$ is the greater of $M_{H1}$ and $M_R$. In this case crabbing governs,

$\therefore M_H = 21.4 \text{ kNm}$

**Note**: When designing the supporting structure it will be found that surge is critical.

Maximum axial force, $N_{sd}$ = maximum design longitudinal load, $W_{h2} = 5.7 \text{ kN}$

### 4.1.3 Summary of design loads

<table>
<thead>
<tr>
<th></th>
<th>$M_{y,SD}$</th>
<th>353.8 kNm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical moment</td>
<td>$M_{x,SD}$</td>
<td>21.4 kNm</td>
</tr>
<tr>
<td>Horizontal moment</td>
<td>$V_{sd}$</td>
<td>232.5 kN</td>
</tr>
<tr>
<td>Vertical shear</td>
<td>$N_{sd}$</td>
<td>5.7 kN</td>
</tr>
</tbody>
</table>
4.2 Cross-sectional properties
The initial sizing of a gantry girder is normally a trial-and-error process. For this example try a 610 × 229 × 125 UB with a 300 mm × 15 mm plate in grade Fe 430 steel (Figure 47). A top flange plate rather than a channel section is adopted because it offers improved access for welding. It should also be noted that many channels will not fit over the flanges of universal beam sections.

![Figure 47 Cross-section through girder (dimensions in mm)](image)

4.2.1 Properties of the plate
Figure 48 shows the axes notation for the plate.

![Figure 48 Axes notation for the plate (dimensions in mm)](image)

\[
\begin{align*}
&b_p = 300 \text{ mm} \\
&t_p = 15 \text{ mm} \\
&A_p = 300 \times 15 \times 10^{-2} = 45 \text{ cm}^2 \\
&I_y = 1/12 \times 300 \times 15^3 \times 10^{-4} = 8.44 \text{ cm}^4 \\
&I_z = 1/12 \times 15 \times 300^3 \times 10^{-4} = 3375 \text{ cm}^4
\end{align*}
\]
4.2.2 Properties of the universal beam

\[
\begin{align*}
    h &= 611.9 \text{ mm} \quad b = 229 \text{ mm} \quad d = 547.3 \text{ mm} \\
    t_e &= 19.6 \text{ mm} \quad t_w = 11.9 \text{ mm} \quad A_b = 160 \text{ cm}^2 \\
    I_y &= 98,600 \text{ cm}^4 \quad W_{dfy} = 3220 \text{ cm}^3 \quad W_{pdfy} = 3680 \text{ cm}^3 \\
    I_z &= 3930 \text{ cm}^4 \quad W_{dfz} = 344 \text{ cm}^3 \quad W_{pdfz} = 536 \text{ cm}^3 \\
    I_i &= 155 \text{ cm}^4
\end{align*}
\]

4.2.3 Calculation of moment of inertia of combined section

The properties of the section may be derived by assuming that it is four rectangles, and tabulating their properties. Table 16 shows the values of the rectangles making up the section. (See Figure 47.)

**Table 16** Properties of rectangles making up the section

<table>
<thead>
<tr>
<th>Rectangle with dimensions (cm)</th>
<th>Area (cm²)</th>
<th>z (cm)</th>
<th>az (cm³)</th>
<th>az² (cm⁵)</th>
<th>I_y,el* (cm⁴)</th>
<th>I_z (cm⁴)</th>
<th>W_{dfy} (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 22.9 x 1.96</td>
<td>44.88</td>
<td>0.98</td>
<td>44</td>
<td>43</td>
<td>14</td>
<td>1961</td>
<td>2177</td>
</tr>
<tr>
<td>2 1.19 x 57.27</td>
<td>68.15</td>
<td>30.60</td>
<td>2085</td>
<td>63 793</td>
<td>18 627</td>
<td>8</td>
<td>1401</td>
</tr>
<tr>
<td>3 22.9 x 1.96</td>
<td>44.88</td>
<td>60.21</td>
<td>2702</td>
<td>162 715</td>
<td>14</td>
<td>1961</td>
<td>481</td>
</tr>
<tr>
<td>4 30 x 1.50</td>
<td>45.00</td>
<td>61.94</td>
<td>2787</td>
<td>172 645</td>
<td>8</td>
<td>3375</td>
<td>560</td>
</tr>
<tr>
<td>Sum</td>
<td>202.92</td>
<td>7619</td>
<td>399 197</td>
<td>18 664</td>
<td>7306</td>
<td>4619</td>
<td></td>
</tr>
</tbody>
</table>

* Where \( I_{y,el} \) is the second moment of area of each of the elements of the cross-section

\[
I_y = \sum I_{y,el} + az^2 \times \bar{y}^2 = 18 664 + 399 197 - 202.92 \times 37.5^2 = 132 504.8
\]

\[
\bar{y} = 37.5 \text{ cm} \quad \text{Depth of eq area axis} = 49.5 \text{ cm}
\]

\[
I_y = 132 504.8 \quad I_z = 7306.0 \text{ cm}^4
\]

\[
i_y = 25.5 \text{ cm} \quad i_z = 6.00 \text{ cm}
\]

\[
W_{df, top} = 5242 \text{ cm}^3 \quad W_{df, bottom} = 3510 \text{ cm}^3
\]

Torsion, \( I_t \) = 181 cm⁴ \quad W_{pdf} = 4619 cm³

4.2.4 Section classification

Steel grade Fe 430, \( t \leq 40 \text{ mm}, \ I_y = 275 \text{ N/mm}^2 \)

\[
\varepsilon = 0.92
\]

- Web (subject to bending)
  
  * For class 2
  
  \[
  \frac{d}{t_w} \leq 83 \varepsilon
  \]

  \[
  \frac{d}{t_w} = \frac{547.3}{11.9} = 46
  \]

  \[
  83 \varepsilon = 76.4
  \]

\[
\therefore \text{ web is at least class 2.}
\]

References

Table 3.1

Table 5.3.1

Table 5.3.1 (Sheet 1)
• Outstand of compound flange compared to thickness of original flange
  (subject to compression)
  \[ \frac{c}{t_f} \leq 11 \varepsilon \]
  \[ \therefore \frac{c}{t_f} = \frac{150}{19.6} = 7.6 \text{ for welded sections} \]
  \[ 11 \varepsilon = 10.2 > 7.6 \]
  \[ \therefore \text{flange is at least class 2.} \]

• Internal width of the added plate between the lines of weld which connect it
to the original flange, compared to its own thickness (subject to compression)
  \[ \frac{b_p}{t_p} \leq 38 \varepsilon \]
  \[ = \frac{229}{15} = 15.3 \]
  \[ 38 \varepsilon = 35.0 > 15.3 \]
  \[ \therefore \text{flange is at least class 2.} \]

• Outstand of the added plate beyond the lines of weld which connect it to
the original flange, compared to its own thickness (subject to compression)
  \[ \frac{b_p - b_h}{1} \leq 11 \varepsilon \text{ for welded sections} \]
  \[ \frac{300 - 229}{15} = 2.4 \]
  \[ 11 \varepsilon = 10.2 > 2.4 \]
  \[ \therefore \text{flange is at least class 2.} \]
4.3 Strength check

4.3.1 Vertical bending resistance

\[ M_{v,y},sd \leq M_{c,y,Rd} \]

\[ M_{c,y,Rd} = \frac{W_{p,y} f_y}{\gamma_{M0}} \quad \text{(class 1 or class 2 cross-sections)} \]

\[ W_{p,y} = 4619 \times 10^3 \text{ mm}^3 \quad \text{(see page 124)} \]

\[ \gamma_{M0} = 1.05 \]

\[ M_{c,y,Rd} = \frac{4619 \times 10^3 \times 275}{1.05 \times 10^6} = 1209 \text{ kNm} \]

\[ M_{c,y,sd} = 353.8 \text{ kNm} \]

\[ ie < 1209 \text{ kNm} \]

\[ \therefore \text{ satisfactory.} \]

4.3.2 Horizontal bending resistance

Horizontal loads are assumed to be carried by the top flange plate only, although the whole of the top flange could be used.

Considering flange plate only:

\[ M_{z,sd} \leq M_{c,z,Rd} \]

\[ M_{c,z,Rd} = \frac{W_{p,z} f_y}{\gamma_{M0}} \quad \text{(class 1 or class 2 cross-sections)} \]

\[ W_{p,z} = \frac{b_p^3 t_p}{4} = \frac{300^3 \times 15}{4} = 337.5 \times 10^3 \text{ mm}^3 \]

\[ \gamma_{M0} = 1.05 \]

\[ M_{c,z,Rd} = \frac{337.5 \times 10^3 \times 275}{1.05 \times 10^6} = 88.4 \text{ kNm} \]

Maximum horizontal moment due to crabbing:

\[ M_{z,sd} = 21.4 \text{ kNm} \]

\[ ie < 88.4 \text{ kNm} \]

\[ \therefore \text{ satisfactory.} \]
4.3.3 Web shear at supports

Maximum design shear \( V_{sd} \leq V_{pcRd} \)

\[
V_{pcRd} = \frac{A_v f_y}{\sqrt{3} \times \gamma_{M0}}
\]

where the shear area \( A_v = 1.04 \times t_w \) for a rolled I section, with the load parallel to the web

\[
A_v = 1.04 \times 611.9 \times 11.9 = 7573 \text{ mm}^2
\]

\[
V_{pcRd} = \frac{7573 \times 275}{\sqrt{3} \times 1.05 \times 10^3} = 1145 \text{ kN}
\]

Maximum design shear \( V_{sd} = 232.5 \text{ kN} \)

\( ie < 1145 \text{ kN} \)

\( \therefore \) satisfactory.

The shear buckling resistance should also be verified if \( d/t_w > 69 \varepsilon \) for an unstiffened web.

**Note** This condition will always be satisfied for grade Fe 430 by the current range of rolled sections. The check is included here for completeness.

\[
d/t_w = \frac{547.3}{11.9} = 46
\]

\( \therefore \varepsilon = 0.924 \)

\[
69 \varepsilon = 69 \times 0.924 = 63.8
\]

\( ie > 46 \)

\( \therefore \) satisfactory.

**Reduction in moment resistance for shear**

Provided that the design value of the shear force \( V_{sd} \) does not exceed 50% of the design plastic shear resistance \( V_{pcRd} \), no reduction need be made in the resistance moments.

\( V_{sd} = 232.5 \text{ kN}, ie > \text{shear at the point of maximum impact,} \)

\( V_{pcRd} = 1145 \text{ kN} \)

\( \therefore V_{sd} < 0.5 \times V_{pcRd} \)

\( \therefore \) no reduction in moment resistance need be made for shear.

4.3.4 Bi-axial bending

In this case the moments \( M_{V, sd} \) and \( M_{h, sd} \) should be multiplied by 0.9 to allow for combining the effects of vertical and horizontal loads.

For class 1 and class 2 cross-sections, as a conservative approximation, the following criterion may be used:

\[
\frac{N_{sd}}{N_{pcRd}} + 0.9 \frac{M_{V, sd}}{M_{pc,Rd}^v} + 0.9 \frac{M_{h, sd}}{M_{pc,Rd}^h} \leq 1
\]

NAD Table 3
\[ N_{sd} = \text{design axial load} = 5.7 \text{ kN} \]
\[ M_{x,sd} = 353.8 \text{ kNm} \quad (\text{see page 122}) \]
\[ M_{z,sd} = 21.4 \text{ kNm} \quad (\text{see page 122}) \]
\[ M_{p_{y,rd}} = 1209 \text{ kNm} \quad (\text{see page 126}) \]
\[ M_{p_{z,rd}} = 88.4 \text{ kNm} \quad (\text{see page 126}) \]

\[
\begin{align*}
\text{ie} & \quad \frac{5.7}{5637.5 + 0.9 \times 353.75/1209 + 0.9 \times 21.38/88.4} = 0.001 + 0.26 + 0.22 \\
& = 0.48 \\
\text{ie} & < 1 \\
\therefore & \text{ satisfactory.}
\end{align*}
\]
4.4 Buckling resistance

4.4.1 Lateral torsional buckling
Check the gantry girder as an unrestrained member under vertical loading. At the supports, the compression flanges are laterally restrained in position and the section is torsionally restrained. However, both the flanges are free to rotate on plan, and there is no warping restraint.

The design buckling resistance moment is given by:

\[ M_{p, Rd} = \frac{\chi_{LT} \beta_w W_p \phi_y f_y}{\gamma_{MI}} \]

where \( \beta_w = 1.0 \) (class 1 or class 2 cross-sections)

\[ \gamma_{MI} = 1.05 \]

\( \chi_{LT} \) is the reduction factor for lateral torsional buckling, obtained from Table 5.5.2, with \( \bar{\lambda} = \lambda_{LT} \) and \( \chi = \chi_{LT} \), from curve a. Note that despite the flange plate being welded to the beam, the section is treated as a rolled section because the welds are remote from the web.

\[ \bar{\lambda}_{LT} = \left( \frac{W_p \phi_y f_y}{M_{cr}} \right)^{0.5} \text{ for } \beta_w = 1 \]

\[ M_{cr} = \text{elastic critical moment for lateral torsional buckling} \]

Because the section is a uniform monosymmetric cross-section with unequal flanges, and is loaded above the shear centre, the following general expression for \( M_{cr} \) given in Annex F must be used:

\[ M_{cr} = \frac{C_1 \pi^2 E I_z}{(k L)^2} \left\{ \left[ \frac{k}{k_w} \right] \frac{I_z}{I_x} + \frac{(k L)^2 G I_x}{\pi^2 E I_z} + \left( C_2 z_a - C_1 z_i \right)^2 \right\}^{0.5} \left( C_2 z_a - C_3 z_i \right) \]

Equation F.2

\( C_1, C_2, \) and \( C_3 \) are factors depending on the loading and end restraint conditions. F.1.2 (1)

\( k \) and \( k_w \) are effective length factors

\[ z_a = z_2 - z_i \]

\( z_2 \) is the co-ordinate of the point of load application

\( z_i \) is the co-ordinate of the shear centre

\[ k = 1.0 \text{ (beam is free to rotate in plan)} \]

\[ k_w = 1.0 \text{ (no special provision for warping fixity)} \]

\( C_1, C_2 \) and \( C_3 \) are taken from Table F.1.2 for a member with two point loads.

For \( k = 1, C_1 = 1.046 \)

\[ C_2 = 0.430 \]

\[ C_3 = 1.120 \]
For an I section with unequal flanges:

\[ I_s = \beta_t (1 - \beta_t) I_z h^2 \]

\[ \beta_t = \frac{I_{fc}}{I_{fc} + I_n} \]

- \( I_{fc} \) = second moment of area of compression flange about minor axis of section = 19.61 + 33.75 = 53.36 \times 10^6 \text{ mm}^4 
  (from Table 16, page 123)
- \( I_n \) = second moment of area of tension flange about minor axis of section = 19.61 \times 10^6 \text{ mm}^4 
  (from Table 16, page 124)
- \( h_s \) = distance between shear centres of flanges

Inserting values into the above equations gives:

\[
\beta_t = \frac{53.36}{53.36 + 19.61} = 0.73
\]

\[
h_s = h + t_p \left( 2 \left[ \frac{b_p^3 + t_p^3}{24} + \frac{b_{tp}^3}{12} \left( t_p + \frac{t_t}{2} \right) \right] - \frac{b_{tp}^3}{12} \right) = 611.9 + 15 - \frac{19.6}{2} - \frac{2293 \times 19.6}{12} 
\]

\[
= 603 \text{ mm}
\]

\[ I_z = I_{zb} + I_{zp} \]

\[ I_{zp} = \frac{t_p b_{zp}^3}{12} = \frac{15 \times 300^3}{12} = 3375 \times 10^4 \text{ mm}^4 \]

\[ I_{zb} = 3930 \times 10^4 \text{ mm}^4 \quad \text{(see page 124)} \]

\[ \therefore I_z = (3375 + 3930) \times 10^4 = 7305 \times 10^4 \text{ mm}^4 \]

\[ \therefore I_w = 0.73 (1 - 0.73) \times 7305 \times 10^4 \times 603^2 \]

\[ = 5.2 \times 10^{12} \text{ mm}^6 \]

Because \( \beta_t > 0.5 \)

\[ z_j = \frac{0.8 (2 \beta_t - 1) \times h_s}{2} \]

\[ = \frac{0.8 (2 \times 0.73 - 1) \times 603}{2} = 110.58 \text{ mm} \]
\[ z_a = z_s - z_g \]
\[ z_a = \text{co-ordinate (relative to centroid) of point of load application} = 611.9 + 15 + \text{crane rail height (h)}_c \]
\[ h_c = 76 \text{ mm} \]
\[ \therefore z_a = 702.9 \text{ mm} \]

\[ z_a = \text{co-ordinate of shear centre (see Figure 49)} \]

For simple span beams with gravity loads

**Figure 49** Location of the shear centre

**Note** For simple span beams with gravity loads, all dimensions are positive above the y-y axis, and the algebraic value of \( z_g \) must be used in the equations.

\[ z_s = \frac{t_s}{2 (1 + v)} = 19.6/2 + (0.73 \times 601) = 448.5 \text{ mm} \]
\[ z_g = 702.9 - 448.5 = 254.4 \text{ mm} \]
\[ I_t = 181 \times 10^4 \text{ mm}^4 \text{ (see Table 16)} \]
\[ G = \frac{E}{2 (1 + v)} \]
\[ v = 0.3 \]
\[ E = 210 000 \text{ N/mm}^2 \]
\[ \therefore G = \frac{210 \times 10^6}{2 \times 1.3} \]
\[ = 80 769 \text{ N/mm}^2 \]

**Equation F.2**

\[ M_{m} = C_1 \frac{k^2 E I_t}{(k L)^2} \left[ \left( \frac{k^2 L}{k w} \right) \frac{I_w}{L} + \frac{(k L)^2 G I_t}{E I_s} + (C_2 z_g - C_3 z_s) \right]^{0.5} - (C_2 z_g - C_3 z_s) \]

where

\[ \frac{C_1 \pi^2 E I_t}{(k L)^2} = \frac{1.046 \times \pi^2 \times 210 000 \times 7306 \times 10^4}{8000^2} \]
\[ = 2.47 \times 10^4 \]

\[ \left( \frac{k^2 L}{k w} \right) \frac{I_w}{L} = \left( \frac{1}{1} \right) \times \frac{5.20 \times 10^{12}}{7305 \times 10^4} \]
\[ = 71 \text{ 184} \]
\[
\frac{(k \, L)^2 \, G \, I_y}{\pi^2 \, E \, I_y} = \frac{8000^2 \times 80 \, 769 \times 181 \times 10^4}{\pi^2 \times 210 \, 000 \times 7306 \times 10^4} = 61 \, 800
\]

\[C_2 \, z_x - C_3 \, z_y = (0.430 \times 254.4) - (1.120 \times 110.58) = -14.46\]

\[\therefore \, M_{cr} = 2.47 \times 10^6 \left[ (71.184 + 61.800 + (-14.46)^2)^{0.5} + 14.46 \right] \times 10^{-6} = 937 \, kNm\]

\[\bar{\lambda}_{LT} = \left( \frac{W_{pL} \, f_y}{M_{cr}} \right)^{0.5} = \left( \frac{4619 \times 10^3 \times 275}{937 \times 10^6} \right)^{0.5} = 1.164\]

For \(\bar{\lambda}_{LT} = 1.164\) and using curve a, then

\[\chi_{LT} = 0.553\]

\[\therefore \, M_{cr, Rd} = \frac{0.553 \times 4619 \times 10^3 \times 275}{1.05 \times 10^6} = 668 \, kNm\]

\[ie > M_{sd} = 353.75 \, kNm, \text{ so the buckling moment resistance is adequate,} \]

\[\therefore \, \text{satisfactory.}\]

4.4.2 Biaxial bending check
In this example the formula is modified by multiplying the moments \(M_{cr, Rd}\) and \(M_{z, Rd}\) by 0.9 to allow for combining the effects of vertical and horizontal loads.

For members for which lateral torsional buckling is a potential failure mode, with zero axial compression and a class 1 or 2 cross-section:

\[
\frac{0.9 \, k_{LT} \, M_{x, Rd}}{\chi_{LT} \, W_{pL} \, f_y} + \frac{0.9 \, k_{z} \, M_{z, Rd}}{\chi_{LT} \, W_{pL} \, f_y} \leq 1
\]

\[
k_{LT} = 1 - \frac{\mu_{LT} \, N_{sd}}{\chi_{LT} \, A \, f_y}
\]

\[
k_{z} = 1 - \frac{\mu_{z} \, N_{sd}}{\chi_{LT} \, A \, f_y}
\]

\[\chi_{LT} = 0.553 \text{ (from above)}\]

Inspection shows that when \(k_{LT}\) and \(k_{z}\) are 1, this equation is similar to the cross-section biaxial bending equation on page 127, except for the \(\chi_{LT}\) term and the partial safety factors \(\gamma_{Mx}\) and \(\gamma_{My}\). However, \(\gamma_{Mx} = \gamma_{My}\) in the NAD. Therefore, this check is always critical if lateral torsional buckling is a potential failure mode.

Substituting the appropriate values from page 128:

\[0.26/0.553 + 0.22 = 0.47 + 0.22 = 0.69\]

\[ie < 1\]

\[\therefore \, \text{satisfactory.}\]
4.5 Web resistance

Resistance of the web to forces applied through a flange is governed by:

- Crushing of the web close to the flange,
- Crippling of the web close to the flange, and
- Buckling of the web over most of the depth of the member.

The crane wheel forces are applied through the top flange and resisted by shear forces in the web. The resistance of the web is the smaller of:

- Crushing resistance, and
- Crippling resistance

4.5.1 Crushing resistance

For wheel loads from cranes, transmitted through a crane rail on a flange but not welded to it, the design crushing resistance of the web, $R_{y,Rd}$, should be taken as:

$$ R_{y,Rd} = \frac{s_y t_w f_{yw}}{\eta_{MI}} $$

$$ s_y = k_R \left( \frac{t_f + \frac{t_k}{t_w}}{1} \right)^{0.33} \times \left[ 1 - \left( \frac{\sigma_{fEd}}{f_{yl}} \right) \right]^{0.5} $$

or, more approximately,

$$ s_y = 2 (h_t + t_i) \left[ 1 - \left( \frac{\sigma_{fEd}}{f_{yl}} \right) \right]^{0.5} $$

Using the second, more approximate, equation:

- $h_t = \text{crane rail height} = 76 \text{ mm (56 kg/m crane rail)}$
- $t_f = \text{combined flange thickness} = 15 + 19.6 = 34.6 \text{ mm}$
- $\sigma_{fEd} = \text{longitudinal stress in flange} = \frac{M_{s,Yd}}{W_{eff}}$
- $W_{eff} = 5242 \times 10^3 \text{ mm}^3$ (see page 124)
- $M_{s,Yd} = 353.8 \text{ kNm}$ (see page 122)

$$ \therefore \sigma_{fEd} = \frac{353.8 \times 10^6}{5242 \times 10^3} = 67.5 \text{ N/mm}^2 $$

$$ \therefore \quad s_y = 2 (76 + 34.6) \left[ 1 - \left( \frac{67.5}{275} \right)^{0.5} \right] = 214 \text{ mm} $$

$$ \therefore \quad R_{y,Rd} = \frac{214 \times 11.9 \times 275}{1.05 \times 10^4} $$

$$ = 667 \text{ kN} $$

The dynamic wheel load is 148.2 kN (see Section 4.1.1).

ie $< 667 \text{ kN}$

$$ \therefore \text{satisfactory.} $$

References

5.7.1 (1)

5.7.1 (3)

5.7.3
4.5.2 Crippling resistance
The stiff bearing length for crane beams may be taken as \( s_y \) given for the web crushing checks,

\[
\therefore s_y = s_y
\]

Crippling resistance, \( R_{a,Rd} = \frac{0.5 t_y^2 (E e_y)^{0.5} [(t_y/t_y)^{0.5} + 3 (t_y/t_y) (s_y/d)]}{\gamma_M} \)

5.7.4 (1)

Figure 50 shows crippling resistance.

\[
s_y/d = 215/547.3 = 0.39 \quad \text{but} \quad 0.2
\]

5.7.4 (1)

\[
s_y \text{ should be reduced to } 0.2 \times 547.3 = 109 \text{ mm.}
\]

\[
\therefore R_{a,Rd} = \frac{0.5 \times 11.9^2 \times (210 \times 275)^{0.5} \times [(34.6/11.9)^{0.5} + 3 \times (11.9/34.6) \times 0.2]}{1.05 \times 10^3}
\]

\[
= 980 \text{ kN}
\]

The factored dynamic wheel load is 148.2 kN (see page 133)

\[
ie < 980 \text{ kN}
\]

\[
\therefore \text{ satisfactory.}
\]

When the member is also subjected to bending moments, the following criterion should be satisfied:

\[
\frac{F_{zd}}{R_{a,Rd}} + \frac{M_{zd}}{M_{e,Rd}} \leq 1.5
\]

where \( F_{zd} = 148.2 \text{ kN} \)

\( M_{zd} = 353.8 \text{ kN} \)

\( R_{a,Rd} = 980 \text{ kN} \)

\( M_{e,Rd} = 1210 \text{ kNm} \)

\[
\therefore \frac{F_{zd}}{R_{a,Rd}} + \frac{M_{zd}}{M_{e,Rd}} = \frac{148.2}{980} + \frac{353.8}{1210} = 0.44
\]

\( ie < 1.5 \)

\[
\therefore \text{ satisfactory.}
\]
4.6 Deflection check
Eurocode 3 currently gives no guidance on limiting deflections for crane girders. Guidance given in British Standard BS 5950 is therefore followed in these deflection checks\(^{13}\).

4.6.1 Vertical deflection
The calculation of the precise deflection of a girder subjected to rolling loads is rather complex. Where two equal rolling loads are applied, a useful assumption is that the maximum deflection occurs at the centre of the beam, with the loads positioned equidistant on either side. This gives the following expression for the maximum deflection resulting from static wheel loads:

\[
\delta_{v, \text{max}} = \frac{W_{s} a_{w} (3 L^2 - a_{w}^2)}{48 E I} \\
= \frac{76 \times 10^3 \times 4000 \times (3 \times 8000^2 - 4000^2)}{48 \times 210000 \times 132556 \times 10^4} \\
= 4.00 \text{ mm}
\]

The limiting vertical deflection resulting from static wheel loads\(^{13}\) is span/600,

\[
\delta_{v, \text{lim}} = \frac{8000}{600} = 13.3 \text{ mm}
\]

\[\therefore \delta_{v, \text{max}} < \delta_{v, \text{lim}}\]

\[\therefore \text{satisfactory.}\]

4.6.2 Horizontal deflection caused by crane surge
Maximum deflection caused by crane surge is given by:

\[
\delta_{H, \text{max}} = \frac{W_{s} a_{w} (3 L^2 - a_{w}^2)}{48 E I}
\]

\[W_{s} = \text{maximum transverse surge load} = 3.0 \text{ kN}\]

\[I_{e} = 3375 \times 10^4 \text{ mm}^4\]

\[\therefore \delta_{H, \text{max}} = \frac{3 \times 10^3 \times 4000 \times (3 \times 8000^2 - 4000^2)}{48 \times 210000 \times 3375 \times 10^4} \\
= 6.21 \text{ mm}
\]

The limiting horizontal deflection caused by crane surge (calculated on the top flange properties alone)\(^{13}\) is span/500.

\[
\delta_{H, \text{lim}} = \frac{8000}{500} = 16.0 \text{ mm}
\]

\[\therefore \delta_{H, \text{max}} < \delta_{H, \text{lim}}\]

\[\therefore \text{satisfactory.}\]

**Note** It is important to ensure that the crane is able to accommodate the deflections of the gantry beam and its supports. It may well not be sufficient simply to check the deflection of the crane beam as an independent member, especially if it is supported on a relatively flexible structure such as a portal frame.
4.7 Design procedure using the concise document (C-EC3)

Most of the gantry girder in the example could be designed using C-EC3. The exceptions are the checks on the lateral torsional buckling moment resistance, in which complexities associated with the destabilising nature of the loading necessitate the use of procedures given in Annex F. For those checks for which C-EC3 is appropriate, the design procedure is similar to that given in Eurocode 3.
Example 5
Stiffened base-plate

5.1 Initial design information
Design a stiffened column base for a 533 x 210 x 122 UB column, as shown in Figure 51, subjected to coexistent design compression, bending moment and shear force.

![Diagram of的设计](image)

Figure 51  General arrangement of column base (dimensions in mm)

5.1.1 Design actions

\[ N_{sd} = 425 \text{ kN} \]
\[ M_{sd} = 290 \text{ kNm} \]
\[ V_{sd} = 45 \text{ kN} \]

Note: The procedure of considering favourable and unfavourable combinations of actions with appropriate load factors to determine the most onerous tensile and compressive loading conditions, has not been included. For clarity, the base has been designed to resist only a single set of design forces. In practice, it would generally need to be rechecked for other conditions by adopting the following procedure for each critical load combination.
5.1.2 Material properties

- Grade Fe 430 steel plate (t < 40 mm) \( f_y = 275 \text{ N/mm}^2 \)  
  Table 3.1
- Holding-down bolts, grade 4.6 \( f_{ub} = 400 \text{ N/mm}^2 \)  
  Table 3.3

Foundation concrete strength class C25/30:

- Characteristic cylinder strength \( f_{ck} = 25 \text{ N/mm}^2 \)  
  Eurocode 2 Part 1
- Characteristic cube strength \( f_{ck,cube} = 30 \text{ N/mm}^2 \)  
  Eurocode 2 Part 1
- Characteristic cube strength of grout = 12 N/mm²
- Partial safety factor for concrete \( \gamma_c = 1.5 \)  
  NAD 6.1.6 a)

5.2 Strength check

In accordance with Annex L.

Bearing strength, \( f_j = \beta_i k_i f_{od} \)  
Annex L.1 (6)

where \( \beta_i = 0.67 \) provided that the characteristic strength of the grout is not less than \( 0.2 f_{ck,cube} \)

ie 12 N/mm² is not less than \( 0.2 \times 30 = 6 \text{ N/mm}^2 \)

\( k_i = 1.0 \) This is a conservative assumption. If the size of the foundation is known, a less conservative value of \( k_j \) (ie \( k_j < 1.0 \)) may be determined. (See the simple base design example in Section 1.12.)

\[ f_{od} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1.5} = 16.7 \text{ N/mm}^2 \]  
Annex L.1 (6)

\[ f_j = 0.67 \times 1.0 \times 16.7 = 11.2 \text{ N/mm}^2 \]  
Annex L.1 (7)

Maximum grout thickness = \( 0.2 \times \) smallest base width

= \( 0.2 \times 440 = 88 \text{ mm} \)

\[ \therefore \text{ use 50 mm thick grout.} \] (For breather holes see Figure 51.)

5.2.1 Effective bearing area

Maximum bearing width, \( c = t \left( \frac{f_y}{3 f_{ub} \gamma_c} \right)^{0.5} \)  
Annex L.1 (3)

\[ = t \left( \frac{275}{3 \times 112 \times 1.05} \right)^{0.5} = 2.8 \text{ t} \]

Adopting a minimum base-plate thickness of 25 mm, ie greater than the column flange thickness (\( t_c = 21.3 \text{ mm} \))  
NAD 6.1.6 a)

\[ c = 2.8 \times 25 = 70 \text{ mm} \]

This results in the reasonably efficient effective bearing area shown in Figure 52.
Effective area under axial load

**Figure 52** Effective bearing area beneath plate (dimensions in mm)

Effective base-plate width, \( b_{\text{eff}} = 4 \times 70 + 2 \times 15 \)

\[ = 310 \text{ mm} \]

Consideration of a rectangular stress distribution in accordance with Eurocode 2 (see Figure 53) produces the following equations:

\[ F_e = 0.8 \times b_{\text{eff}} f' \]  

(Equation 1)

\[ F_t = F_e - N_{\text{sd}} \]  

(Equation 2)

Moments about \( T \):

\[ N_{\text{sd}} \left( \frac{a}{2} - (a - d_1) \right) \]

\[ N_{\text{sd}} \left( \frac{a}{2} - a + d_1 \right) \]

\[ N_{\text{sd}} (d_1 - \frac{a}{2}) \]

**Figure 53** Plastic distribution of actions

**Note** In Eurocode 2, \( x \) is the distance to the plastic neutral axis, the depth of the stress block being 0.8 \( x \).
Taking moments about the tension bolt centre line:

\[ M_{sd} = F_e (d_i - 0.4 \, x) - \frac{N_{sd}}{2} \times (2 \, d_i - a) \]  \hspace{1cm} (Equation 3)

Substituting \( F_e \) from Equation 1:

\[ M_{sd} = 0.8 \times d_i \times b_{ef} \times 0.32 \times x^2 \times b_{ef} \times \frac{N_{sd}}{2} \times (2 \, d_i - a) \]  \hspace{1cm} (Equation 4)

Substitute in Equation 4 and solve for \( x \):

\[ \therefore \quad 290 \times 10^6 = 0.8 \times 1075 \times 310 \times 11.2 - 0.32 \times x^2 \times 310 \times 11.2 - (425 \times 10^6)/2 \times (2 \times 1075 - 1150) \]

\[ \therefore \quad 1111 \times x^2 - 2.96 \times 10^6 + 502.5 \times 10^6 = 0 \]

\[ \therefore \quad x^2 = 2.66 \times 10^5 + 452.3 \times 10^3 = 0 \]

\[ \therefore \quad x = \frac{2660 \pm \sqrt{(2660^2 - 4 \times 452.3 \times 10^3)}}{2} \]

\[ = 1330 \pm 1147 = 183 \text{ mm} \]

Substitute in Equation 1 for \( x \):

\[ F_e = 0.8 \times 183 \times 310 \times 11.2 \times 10^{-3} = 508 \text{ kN} \]

\[ \therefore \quad F_e = 508 - 425 = 83 \text{ kN} \]

\[ 5.2.2 \quad \textbf{Check adequacy of holding-down bolts} \]

| Tension/Bolt | 83/2 | 41.5 kN |

**Note** Eurocode 3 requires that prying forces, where they are necessary for equilibrium, must also be taken into account. In this case the base-plate is sufficiently thick and the bolts are positioned sufficiently close to the gusset plates for the base-plate to be designed assuming cantilever action of the plate as opposed to bending in double curvature. Consequently, prying forces are not necessary to maintain equilibrium and do not, therefore, have to be considered in the design of the bolts.

Try M24 grade 4.6 HD bolts

\[ F_{t,Rd} = \frac{0.9 \, f_{sb} \, A_s}{\gamma_{Mb}} \]

where \( \gamma_{Mb} = 1.35 \)

\[ = \frac{0.9 \times 400 \times 353}{1.35 \times 10^3} = 94 \text{ kN} \]

\( \therefore \) satisfactory.

\( \therefore \) use four M24 grade 4.6 square-headed HD bolts complying with British Standard BS 7419\(^{17}\).
5.2.3 Base-plate bending in the tension zone

As prying forces were not considered in the design of the bolts, the base-plate is designed in single curvature. Moment from face of gusset plate:

\[ M_{sd} = \frac{F_t}{2} \times 54 = 41.5 \times 54 \times 10^{-3} = 2.2 \text{ kNm} \]

Consider the effective length of the end-plate resisting bending using the design philosophy from Annex J.

\[
\begin{align*}
  m &= 54 - 0.8 (a \times \sqrt{2}) \\
      &= 54 - 0.8 \times 6 \times \sqrt{2} \\
      &= 47.2 \text{ mm} \\
  n &= e_{\text{min}} \leq 1.25 \text{ m} \\
      &= 45 \text{ mm} < 59.0 \text{ mm} \\
      &= 45 \text{ mm}
\end{align*}
\]
The basic yield line pattern shown in Figure J.3.4 (b) in Eurocode 3 has been modified to be representative of a bolt which is bounded by two adjacent discontinuous edges. It is evident from Figure 54 that the effective length of the yield line is equivalent to half the value in clause J.3.4.1(2)b plus an end distance.

![Diagram of yield line pattern]

\[ l_{\text{eff}} = \frac{1}{2} (4m + 1.25e) + 75 \]

\[ = 2m + 0.625e + 75 \]

\[ = 2 \times 47.2 + 0.625 \times 45 + 75 \]

\[ = 123 + 75 \]

\[ = 198 \text{ mm} \]

**Figure 54** Assumed yield line pattern (dimensions in mm)
Resistance moment:

\[ M_{Pr,dl} = \frac{f_{\text{eff}} r^2 f_y}{4 \gamma_{M0}} \]

\[ = \frac{201 \times 25^2 \times 275}{4 \times 1.05 \times 10^6} \]

\[ = 8.2 \text{ kNm} \]

\[ \therefore \text{ie} > 2.2 \text{ kNm} \]

\[ \therefore 25 \text{ mm thick grade Fe 430A base-plate.} \]

**Note** It is assumed that the service condition corresponds to that of an internal environment (SI) with a minimum service temperature of \(-5^\circ\text{C}\). Therefore, the NAD permits a steel quality designation 'A'.

---

**5.2.4 Horizontal shear**

Where the applied shear force is less than 20% of the applied vertical load, the shear load can be transferred from the base-plate to the foundation by friction[^1].

\[ N_{sd} = 425 \text{ kN} \quad \therefore 0.2 \times N_{sd} = 85 \text{ kN} \]

\[ V_{sd} = 45 \text{ kN} \quad \text{ie} \quad V_{sd} < 85 \text{ kN} \]

\[ \therefore \text{satisfactory.} \]

---

**5.2.5 Cheek-plates**

(Figure 55)

Moment about the column face:

\[ M_{sd} = \frac{508 \times 230}{2 \times 1000} = 58.4 \text{ kNm/plate} \]

\[ V_{sd} = \frac{508}{2} = 254 \text{ kNm/plate} \]

![Figure 55](image_url)  

External forces on cheek-plates (dimensions in mm)
All base-plate stiffeners should be designed so that the elastic limit is not exceeded (to be consistent with Annex L.1 (2)). Adopting a 15 mm thick plate and assuming this will be class 3 (to be checked later), determine the depth required by considering the elastic modulus.

**Note** It is assumed that the cheek-plates resist the applied moment without acting in conjunction with the base-plate, in other words, in isolation as opposed to an inverted tee. This gives a conservative estimate of the plate required. If advantage is taken of both plates acting together, this must be taken into account in the design of the welds between the stiffener and the base-plate. Here the base-plate is considered as a separate slab supported by the stiffener.

\[
d_{\text{required}} = \sqrt{(6 M_{\text{sd}}/t f_y)} = \sqrt{(6 \times 58.4 \times 10^6 \times 1.05)/(15 \times 275)} \\
= 299 \text{ mm}
\]

Try a 300 mm deep cheek-plate.

Check the shear resistance:

\[
V_{p\text{Rd}} = \frac{A_s f_y}{\sqrt{3} \gamma_{Mc}} = \frac{15 \times 300 \times 275}{\sqrt{3} \times 1.05 \times 10^3} \\
= 680 \text{ kN}
\]

\[i.e \quad > 252.5 \text{ kN}\]

\[\therefore \text{ satisfactory.}\]

**Note** \[V_{sd} < 0.5 V_{p\text{Rd}} = 0.5 \times 680 = 340 \text{ kN}\]  

\[\therefore \text{ no reduction of } M_{p\text{Rd}} \text{ as a result of the effect of the applied shear.}\]

Check the check-plate for local buckling as a class 3 element.

**Note** The local buckling check has been performed by applying the slenderness criteria for general sections given in Eurocode 3. When applied to check, gusset or stiffening plates in connections, these rules are very conservative because there are two stiffened edges at right angles. Slenderness criteria specifically for such connection plate elements can be found in *Structural steelwork connections*14.

For welded outstand (compression):

\[
c/t = 14 \ \varepsilon \\
c = 14 \times 0.92 \times 15 \\
= 193 \text{ mm}
\]

Table 5.3.1 (Sheet 3)
Assume that the local buckling requirements are satisfied if the intersection point as shown in Figure 56 is outside the check-plate.

![Figure 56](image)

Figure 56  Local buckling criterion for check-plate (dimensions in mm)

Adopt 300 mm deep × 15 mm thick check-plates (provided that this is deep enough for a satisfactory weld between the check-plate and the column flange).

### 5.2.6 Weld requirements

**Check-plates to column flange**

\[ V_{sd} = \frac{N_{sd}}{2} + \frac{M_{sd}}{d} \]

\[ = \frac{425}{2} + \frac{290 \times 10^3}{523} = 745 \text{ kN} \]

Shear/check-plate = \( \frac{745}{2} = 372.5 \text{ kN} \)

Shear/unit length of weld (2 welds) = \( \frac{372.5}{2 \times 300} = 0.62 \text{ kN/mm} \)

Design shear strength of weld \( (f_{sw}) \)

\[ = \frac{f_{sv} \sqrt{3}}{\beta_w \chi_{sw}} = \frac{430/\sqrt{3}}{0.85 \times 1.35} = 216.3 \text{ N/mm}^2 \]

Leg length required = \( \frac{V_{sd}}{0.7 f_{sw}} = \frac{0.62 \times 10^3}{0.7 \times 216.3} = 4.1 \text{ mm} \)

but use 8 mm for welding to this thickness of flange and base-plate.

**Check-plate to base-plate**

Shear/unit length of weld:

Compression zone  Section is to be in direct bearing. Welding requirements are therefore nominal.

\[ Tension \text{ zone } = \frac{F_t}{4 \ell_{eff}} = \frac{76}{4 \times 201} = 0.1 \text{ kN/mm} \]

Leg length required = \( \frac{V_{sd}}{0.7 f_{sw}} = \frac{0.1 \times 10^3}{0.7 \times 216.3} = 0.66 \text{ mm} \)
Although the base-plate was not included in the design of the cheek-plate they may act compositely, so the welds between the two will be checked, assuming full interaction. The moment of inertia of the two is determined from Table 17

**Table 17  Determination of moment of inertia**

<table>
<thead>
<tr>
<th></th>
<th>A (cm²)</th>
<th>y (cm)</th>
<th>Ay (cm³)</th>
<th>Ay² (cm⁵)</th>
<th>I (cm⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>38.75</td>
<td>1.25</td>
<td>48.44</td>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>Stem</td>
<td>45.00</td>
<td>17.5</td>
<td>787.5</td>
<td>13 781</td>
<td>3375</td>
</tr>
<tr>
<td>Total</td>
<td>83.75</td>
<td>836</td>
<td>13 841</td>
<td>33 95</td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{836}{83.75} = 10.3 \text{ cm} \]
\[ I_s = 3395 + 13 841 - 83.75 \times 10.3^2 = 8893 \text{ cm}^4 \]

Shear on weld = \[ \frac{SA \bar{y}}{I_s} = \frac{252.5 \times 45 \times 10^3 \times 9.05 \times 10}{8893 \times 10^4} \]
= 1.16 kN/mm

Leg length required = \[ \frac{1.16 \times 10^3}{0.7 \times 208.6} \]
= 7.9 mm

This check governs the weld design.

\[
\therefore \text{ use a nominal 8 mm fillet weld both sides of cheek-plates. The weld will be made continuous to form a seal against corrosion.}
\]
5.3 Design procedure using the concise document (C-EC3)

With the exception of the base-plate appraisal in Annex J of Eurocode 3, this example can also be designed using C-EC3. The design is similar to that in the Eurocode itself, except for the following check.

Weld: cheek-plates to column flange

In C-EC3 the calculation of $f_{vw}$ has been replaced by a table of design strengths.

Shear/cheek-plate $= \frac{745}{2} = 372.5 \text{ kN}$
Shear/unit length of weld ($V_{sb}$) $= \frac{372.5}{300} = 1.24 \text{ kN/mm}$

Using grade Fe 430 steel,
Design resistance of weld ($F_{vw}$) $= 216 \text{ N/mm}^2$

Leg length required $= \frac{V_{sb}}{0.7 f_{vw}} = \frac{1.24 \times 10^3}{0.7 \times 196 \times 2}$

$= 4.1 \text{ mm}$

\[.\text{ use an 8 mm continuous fillet weld for cheek-plate/flange vertically and for base-plate/flange horizontally.} \]
6.1 Initial design information

Design the beam splice shown in Figure 57 to carry a design bending moment of 135 kNm and a design shear force of 220 kN.

![Beam splice diagram](image)

*Figure 57* Beam splice details (dimensions in mm)

It is assumed that the moment is carried solely by the flange plates, with the shear carried by the web plates.

The designer looking for economy should consider using bolts in clearance holes. They will generally cost less than HSFG bolting. If bolts in clearance holes are used, there will be movement in the joint as the bolts slip into bearing, and consequently less stiffness in the member. These bolts should be used only if such considerations have no detrimental effect on the design assumptions or the overall structural response.

For this example it is assumed that deformations caused by slip are unacceptable right up to ultimate load, therefore HSFG bolts (category C) will be used, as they are designed to prevent slip at ultimate load. A more normal assumption could be category B, no slip at serviceability.

6.1.1 Design criteria

For category C connections (slip resistant under ultimate load) there are two main design criteria:

\[
F_{v,Sl} \leq F_{v,Rd} \quad \text{(applied bolt shear less than design slip resistance)}
\]

\[
F_{v,Sl} \leq F_{b,Rd} \quad \text{(applied bolt shear less than design bearing resistance)}
\]
6.2 Web splice

The plate on the sides of the web, and the bolts, will be designed to transmit the whole of the vertical shear of 220 kN.

Bolt forces:
Vertical shear/bolt, \( V_{cl} = \frac{V_{cl}}{n} = \frac{220}{4} = 55 \text{ kN} \)

In addition to the shear, there is a local moment due to the eccentricity of the bolt lines. The moment will be shared equally between the two bolt lines.

\[
M_{cl} = \frac{220 \times 85}{2 \times 10^3} = 9.35 \text{ kNm}
\]

Assuming a linear distribution (see Figure 58)

\[
F_{h,cl} = \frac{6 M_{cl}}{n(n+1) \eta} = \frac{9.35 \times 10^3 \times 6}{4(4+1) \times 90} = 31.2 \text{ kN}
\]

![Figure 58](Image)

Figure 58 Distribution of bolt forces (dimensions in mm)

Resultant shear force on outer bolts

\[
F_{res,cl} = \left[ (F_{h,cl})^2 + (F_{v,cl})^2 \right]^{0.5} = \left[ 31.2^2 + 55^2 \right]^{0.5} = 63.2 \text{ kN}
\]

6.2.1 Slip resistance of HSFG bolts

In this example, the surface treatment of the steel corresponds to class B

\[ \therefore \mu = 0.4 \]

Note Class B corresponds to a surface which is cleaned by grit or shot blasting and painted with an alkali-zinc silicate paint.

Design preload force assuming M20 general grade bolts

\[ F_{p,cl} = 144 \text{ kN} \]

References

Figure 6.5.7 (a)
6.5.4 (1)

6.5.8.3 (3)
6.5.8.3 (1)

NAD 6.1.4 d)
Design slip resistance

\[ F_{Srd} = \frac{k_s n \mu}{\gamma_{Ms}} \times F_{p,Cd} \]

where \( k_s = 1.0 \) for bolts in standard clearance holes
\( n = 2 \) (two cover plates, \( \therefore \) two faying surfaces)
\( \gamma_{Ms} = \gamma_{Msalt} = 1.2 \) (category C)
\[ = \frac{1.0 \times 2 \times 0.4}{1.2} \times 144 = 96 \text{ kN} \]

ie \( > 63.2 \) kN
\( \therefore \) satisfactory.

### 6.2.2 Bearing resistance of HSFG bolts

**Note** Although the connection is designed to be slip resistant under ultimate loads, it is a requirement that the bolts have adequate bearing resistance. However, it is not necessary to limit the deformation in this case.

End distances:
- Beam web \( e_1 = 40 \times 63.2 / 31.2 = 81 \) mm
- Cover plate \( e_1 = 40 \times 63.2 / 55 = 46 \) mm

**Note** The end distances are those in the direction of the resultant of the horizontal and vertical load components (see Figure 59).

![Web plate](image)

**Figure 59** Web plate end distances (dimensions in mm)

\[ F_{b, Rd} = \frac{2.5 \alpha f_u d t}{\gamma_{Mb}} \]

where \( \alpha \) is the smallest of:

*End bolt tearout in:*

- Cover plate \( \alpha = e_1 / 3 \, d_o = \frac{46}{3 \times 22} = 0.70 \)
- Beam web \( \alpha = e_1 / 3 \, d_o = \frac{81}{3 \times 22} = 1.23 \)
Tearing between bolt rows:

\[ \frac{P_t}{3d_o - 0.25} = \frac{90}{3 \times 22} - 0.25 = 1.1 \]

Bearing failure of bolt:

\[ f_{tb}/f_u = 800/430 = 1.86 \]

Bearing failure of plate

\[ \alpha = 1.0 \]

Cover plates (try two, 8 mm thick) \(-\alpha_{\text{min}} = 0.7\)

\[ F_{b,\text{Rd}} = \frac{2 \times 2.5 \times 0.7 \times 430 \times 20 \times 8}{1.35 \times 10^3} = 178 \text{ kN} > 63.2 \text{ kN} \]

Beam web \(-\alpha_{\text{min}} = 1.0\)

\[ F_{b,\text{Rd}} = \frac{2.5 \times 1.0 \times 430 \times 20 \times 8.5}{1.35 \times 10^3} = 135 \text{ kN} > 63.2 \text{ kN} \]

\[ \therefore \text{ satisfactory.} \]

6.2.3 Shear resistance of beam web

The shear resistance of the beam web is:

\[ V_{p(Rd)} = A_v \times \frac{f_v \times \sqrt{3}}{\gamma_{M0}} \]

\[ A_v = 1.04 \times t_w = 1.04 \times 453.6 \times 8.5 = 4010 \text{ mm}^2 \]

\[ A_{v,\text{net}} = 4010 - 4 \times 22 \times 8.5 = 3262 \text{ mm}^2 \]

\[ f_v/f_u = 275/430 = 0.64 \]

\[ A_{v,\text{net}}/A_v = 3262/4010 = 0.81 > 0.64 \]

\[ \therefore \text{ use gross shear area.} \]

\[ V_{p(Rd)} = A_v \times f_v = \frac{4010 \times 275}{\sqrt{3} \times 1.05 \times 10^3} = 606 \text{ kN} \]

\[ i_e > 220 \text{ kN} \]

\[ \therefore \text{ satisfactory.} \]
6.2.4 Shear rupture resistance

This check ensures that the presence of bolt holes in a confined area will not precipitate a local 'block' failure of the section (see Figure 60).

![Figure 60 Block failure of the web (dimensions in mm)](image)

The effective resistance is given by:

\[ V_{\text{eff},Rd} = (f_y/\sqrt{3}) A_{\text{veff}}/y_{M0} \]

where

\[ A_{\text{veff}} = t L_{\text{veff}} \]

- \( t \) is the web thickness
- \( L_{\text{veff}} = L_v + L_1 + L_2 \) but less than \( L_3 \)
- \( L_v \) is shown in Figure 60
- \( L_1 \) is \( a_1 \) but \( \leq 5 \, d \) (\( 5 \, d = 110 \))
- \( L_2 \) is \( (a_2 - k \, d_{c,x})(f_y/f_{c,y}) \)
- \( L_3 = L_v + a_1 + a_3 \) but \( \leq (L_v + a_1 + a_3 - n \, d_{c,x})(f_y/f_{c,y}) \)
- \( k = 0.5 \) for a single row of bolts

Example values
(Figure 60)

\[ L_{\text{veff}} = 270 + 91.8 + (40 - 0.5 \times 22)(430/275) = 407 \, \text{mm} \]

Limiting \( L_3 = 270 + 91.8 + 91.8 = 453.6 \)

limited to \( (270 + 91.8 + 91.8 - 4 \times 22) \times (430/275) = 578 \)

\[ L_{\text{veff}} = 407 \, \text{mm} \]

\[ V_{\text{eff},Rd} = (275/\sqrt{3}) \times \frac{8.5}{1.05 \times 10^3} = 523 \, \text{kN} \]

\[ \therefore 220 \, \text{kN}, \]

\[ \therefore \text{ satisfactory.} \]

6.2.5 Shear resistance of web cover plates

Inspection shows that the shear resistance of the web cover plates will be satisfactory as they are of a similar thickness to the beam web and subject to only half the loading.

In-plane bending of the cover plates is also considered not to be significant, for similar reasons.

\[ \therefore \text{ satisfactory.} \]
6.3 Flange splice

Force at the flange/cover plate interface:

\[ F_{h,SD} = \frac{M_g}{h} = 135 \times 10^6/453.6 = 298 \text{ kN} \]

\[ F_{s,RL} = \text{shear resistance of grade 8.8, 20 mm bolts} = 48.0 \text{ kN} \]

Number of M20 general grade HSFG bolts required

\[ = \frac{F_{h,SD}}{F_{s,RL}} = \frac{298}{48.0} = 6.21 \]

\[ \therefore \text{ provide eight bolts each side of the splice.} \]

6.3.1 Bearing resistance

Check bearing (see Figure 61).

Horizontal shear/bolt \( = F_{h,SD} = 298/8 = 37.3 \text{ kN} \)

![Figure 61 Bearing resistance of flange splice (dimensions in mm)](image)

It is assumed that the splice plate is thicker than the flange, so the flange thickness should be used to determine the limiting bearing resistance.

\[ F_{h,RL} = \frac{2.5 \alpha f_{u} d t}{\gamma_{Mb}} \]

where \( \alpha \) is the smallest value from the following:

\[ e_t / d_o = \frac{40}{3 \times 22} = 0.61 \]

\[ p_t / d_o - 0.25 = \frac{70}{3 \times 22} - 0.25 = 0.81 \]

\[ f_{u} / f_{y} = 1.86 \]

\[ \alpha_{min} = 0.61 \text{ (end-bolt tearout)} \]

\[ F_{h,RL} = \frac{2.5 \alpha f_{u} d t}{\gamma_{Mb}} = \frac{2.5 \times 0.61 \times 430 \times 20 \times 12.7}{1.2 \times 10^3} \]

\[ = 139 \text{ kN} \]

\[ ic > 37.3 \text{ kN} \]

\[ \therefore \text{ satisfactory.} \]

\[ \therefore \text{ use 16 M20 general grade HSFG bolts for flange splices.} \]
6.3.2 Check beam flange in tension

\[ A = b t_f = 189.9 \times 12.7 = 2412 \text{ mm}^2 \]

\[ A_{\text{net}} = (b t_f - n d_p) = 189.9 \times 12.7 - 2 \times 22 \times 12.7 = 1853 \text{ mm}^2 \]

Design resistance of the net section at bolt holes of category C connections

\[ N_{\text{net, Rd}} = \frac{A_{\text{net}} f_y}{\gamma_{MO}} = \frac{1853 \times 275}{1.05 \times 10^3} = 485 \text{ kN} \]

Design ultimate resistance of the net section at bolt holes.

\[ N_{u, Rd} = \frac{0.9 A_{\text{net}} f_y}{\gamma_{M2}} = \frac{0.9 \times 1853 \times 430}{1.2 \times 10^3} = 597 \text{ kN} \]

\( N_{\text{net, Rd}} \) gives critical value of 485 kN

\( i_e > 298 \text{ kN} \)

\( \therefore \) satisfactory.

6.3.3 Flange cover plates in tension

An adequate plate thickness will normally be assured if the next standard plate thickness above the beam flange thickness is selected, and provided that the plate is approximately the same width as the beam. Frequently, the plate is assumed to have the same area as the beam flange.

\( \therefore \) use 180 \times 15 \text{ mm thick grade Fe 430 flange plates.} \)
6.4 Design procedure using the concise document (C-EC3)

This example can also be designed using the concise version of the Eurocode. The procedure is essentially the same, except for the following check which is much simpler.

**Bearing resistance**

Flange cover plates:

Edge distance = 40 mm = 2d

Pitch = 70 mm = 3.5d

\[ \therefore \text{bearing strength coefficient, } \beta = 0.64 \text{ kN/mm}^2 \]

**Note** As the bolts are designed to be non-slip at ultimate load, \( \beta \) is determined for the case where limiting the deformation is not necessary.

\[ F_{b,Rd} = \beta d t \gamma_{Mb} \]

\[ \gamma_{Mb} = 1.35 \]

\[ d = 20 \text{ mm} \]

\[ t = 12.7 \text{ mm} \]

\[ \therefore F_{b,Rd} = \frac{0.64 \times 20 \times 12.7}{1.35} = 120.4 \text{ kN} \]

\[ \text{ie } F_{v,SD} (37.3 \text{ kN}) \]

\[ \therefore \text{ satisfactory.} \]
References


This book is for engineers and students designing steel structures to Eurocode 3.


The examples are a 5-storey steel-framed building, a 2-storey rigid frame, a 30 m span roof truss, a gantry girder to support a 100 kN capacity crane, a stiffened base-plate and an beam splice. Each example has been prepared to give a detailed indication of the process of designing steel structures to Eurocode 3, including all the checks required by the Eurocode and the UK National Application Document.